Computer algebra independent integration tests

3-Logarithms/3.2.3-u-log-e-f-a+b-x-^p-c+d-x-^q-^r-^s

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [108]. This is test number [61].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.15 (106)	% 1.85 (2)
Mathematica	% 100.00 (108)	% 0.00 (0)
Maple	% 22.22 (24)	% 77.78 (84)
Maxima	% 62.96 (68)	% 37.04 (40)
Fricas	% 37.04 (40)	% 62.96 (68)
Sympy	% 20.37 (22)	% 79.63 (86)
Giac	% 31.48 (34)	% 68.52 (74)
Mupad	% 32.41 (35)	% 67.59 (73)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

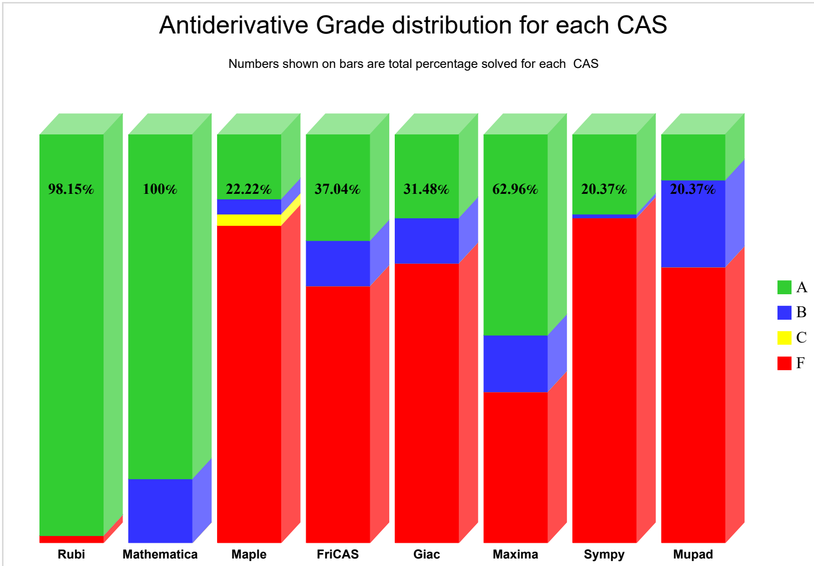
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

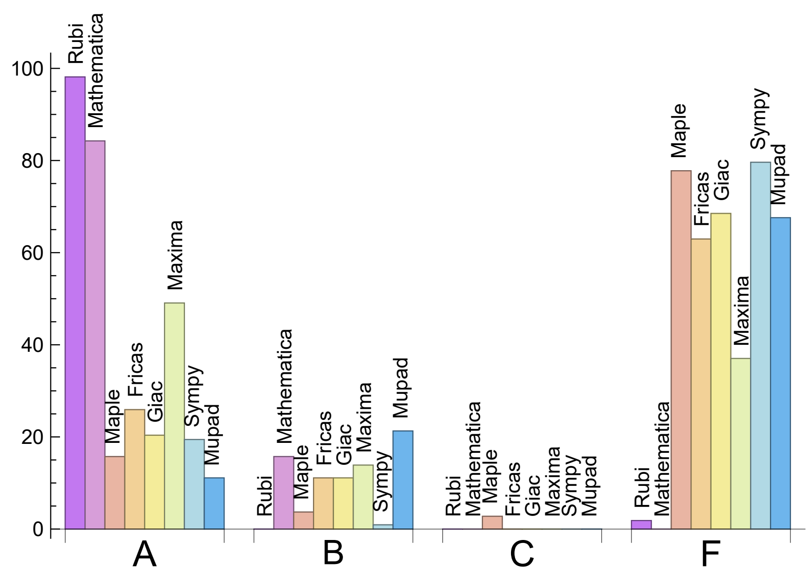
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.15	0.00	0.00	1.85
Mathematica	84.26	15.74	0.00	0.00
Maple	15.74	3.70	2.78	77.78
Maxima	49.07	13.89	0.00	37.04
Fricas	25.93	11.11	0.00	62.96
Sympy	19.44	0.93	0.00	79.63
Giac	20.37	11.11	0.00	68.52
Mupad	11.11	21.30	0.00	67.59

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	84	98.81 %	1.19 %	0.00 %
Maxima	40	75.00 %	0.00 %	25.00 %
Fricas	68	95.59 %	4.41 %	0.00 %
Sympy	86	37.21 %	61.63 %	1.16 %
Giac	74	63.51 %	36.49 %	0.00 %
Mupad	73	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

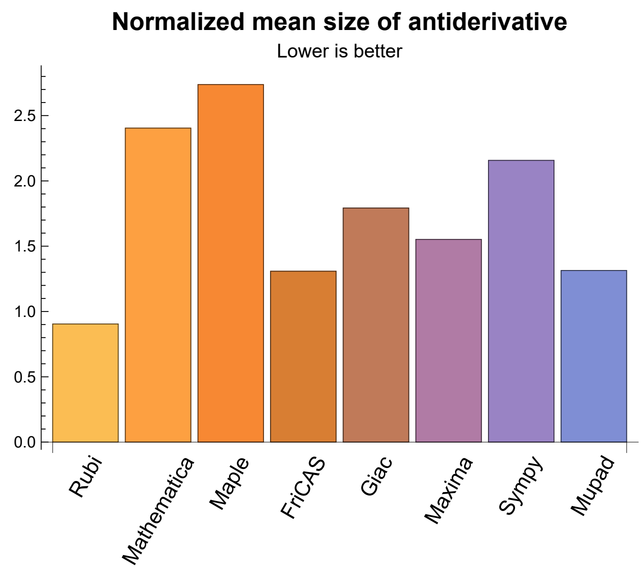
1.3 Performance

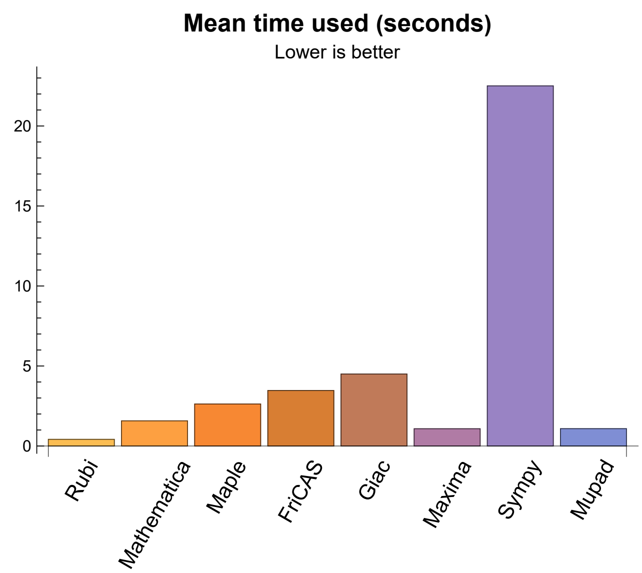
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.42	350.24	0.90	182.50	1.00
Mathematica	1.57	1434.26	2.40	165.00	0.96
Maple	2.62	332.08	2.74	32.50	1.01
Maxima	1.08	430.15	1.55	164.50	1.23
Fricas	3.47	170.18	1.31	48.00	1.22
Sympy	22.51	170.64	2.16	44.00	0.98
Giac	4.50	304.35	1.79	62.00	1.13
Mupad	1.08	261.03	1.31	44.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

$\{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {67, 68}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

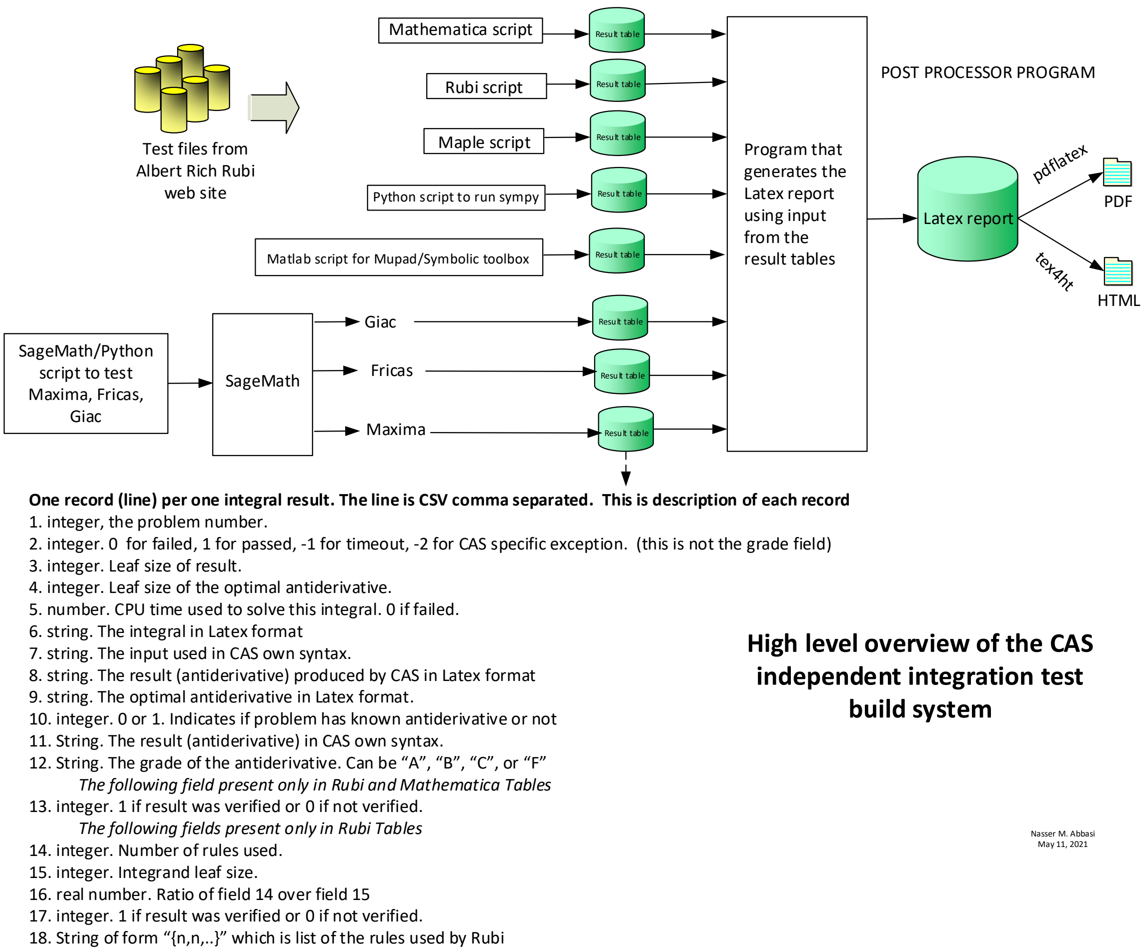
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade: { }

C grade: { }

F grade: { 74, 75 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade: { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 67, 68, 69, 92, 106, 107, 108 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 29, 54, 55, 60, 61, 62, 63, 65, 66, 70, 72, 73, 89, 93, 96, 102, 106 }

B grade: { 88, 99, 104, 107 }

C grade: { 64, 74, 75 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 67, 68, 69, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 94, 95, 97, 98, 100, 101, 103, 105, 108 }

2.1.4 Maxima

A grade: { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 54, 55, 59, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 79, 93, 94, 96, 97, 99, 100 }

B grade: { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 77, 81, 88, 89 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 64, 67, 68, 69, 76, 78, 80, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.5 FriCAS

A grade: { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 89, 93, 96, 99 }

B grade: { 7, 8, 9, 13, 14, 15, 25, 26, 27, 31, 44, 45 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.6 Sympy

A grade: { 10, 12, 28, 29, 43, 44, 45, 46, 47, 62, 63, 65, 66, 72, 73, 74, 75, 89, 93, 96, 99 }

B grade: { 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.7 Giac

A grade: { 10, 12, 13, 27, 29, 31, 47, 48, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 96, 99 }

B grade: { 7, 8, 9, 14, 15, 32, 33, 34, 46, 49, 50, 93 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.8 Mupad

A grade: { 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73 }

B grade: { 7, 8, 9, 10, 12, 13, 14, 15, 25, 26, 27, 28, 29, 31, 32, 33, 34, 74, 75, 89, 93, 96, 99 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	385	336	0	0	0	0	0	-1
normalized size	1	0.95	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	0.409	0.115	0.000	0.427	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	242	217	0	0	0	0	0	-1
normalized size	1	0.92	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.214	0.070	0.000	0.423	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.096	0.077	0.000	0.451	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.155	0.218	0.000	0.425	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	352	295	0	0	0	0	0	-1
normalized size	1	1.09	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.478	0.069	0.000	0.430	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	562	470	0	0	0	0	0	-1
normalized size	1	1.06	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.797	1.225	0.072	0.000	0.419	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	185	0	395	624	0	560	886
normalized size	1	1.00	0.92	0.00	1.97	3.10	0.00	2.79	4.41
time (sec)	N/A	0.095	0.308	1.076	0.739	0.434	0.000	34.441	0.697

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	154	0	285	469	0	407	501
normalized size	1	1.00	0.90	0.00	1.66	2.73	0.00	2.37	2.91
time (sec)	N/A	0.072	0.215	0.305	0.907	0.441	0.000	8.961	0.533

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	127	0	194	325	0	269	255
normalized size	1	1.00	0.89	0.00	1.36	2.27	0.00	1.88	1.78
time (sec)	N/A	0.061	0.138	0.293	0.654	0.433	0.000	2.355	0.469

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	105	0	118	197	427	148	128
normalized size	1	1.00	0.91	0.00	1.02	1.70	3.68	1.28	1.10
time (sec)	N/A	0.043	0.200	0.156	0.682	0.420	60.278	0.713	0.401

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	164	0	0	0	-1
normalized size	1	1.00	0.87	0.00	1.53	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.108	0.327	0.801	0.425	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	0	99	120	1931	112	99
normalized size	1	1.00	0.94	0.00	1.04	1.26	20.33	1.18	1.04
time (sec)	N/A	0.037	0.055	0.305	0.710	0.427	139.980	0.192	2.160

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	0	165	323	0	246	182
normalized size	1	1.00	0.86	0.00	1.22	2.39	0.00	1.82	1.35
time (sec)	N/A	0.056	0.243	0.313	0.748	0.440	0.000	0.225	2.389

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	141	0	289	580	0	469	346
normalized size	1	1.00	0.86	0.00	1.76	3.54	0.00	2.86	2.11
time (sec)	N/A	0.069	0.358	0.307	0.735	0.438	0.000	0.231	2.869

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	164	0	459	861	0	748	526
normalized size	1	1.00	0.85	0.00	2.38	4.46	0.00	3.88	2.73
time (sec)	N/A	0.085	0.357	0.306	0.848	0.441	0.000	0.206	3.712

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	920	920	2508	0	1421	0	0	0	-1
normalized size	1	1.00	2.73	0.00	1.54	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.845	2.867	0.316	1.099	0.500	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	1853	0	1071	0	0	0	-1
normalized size	1	1.00	2.30	0.00	1.33	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	2.086	0.307	1.192	0.481	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	1211	0	769	0	0	0	-1
normalized size	1	1.00	1.77	0.00	1.12	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	1.287	0.300	0.879	0.457	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	781	0	504	0	0	0	-1
normalized size	1	1.00	1.45	0.00	0.93	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	0.677	0.151	1.130	0.426	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	460	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.203	0.311	0.000	0.410	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	411	0	392	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	0.710	0.310	0.895	0.455	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	872	0	755	0	0	0	-1
normalized size	1	1.00	1.38	0.00	1.19	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	1.329	0.312	1.098	0.471	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	1407	0	1252	0	0	0	-1
normalized size	1	1.00	1.84	0.00	1.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	1.945	0.309	1.727	0.536	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	884	884	2003	0	1816	0	0	0	-1
normalized size	1	1.00	2.27	0.00	2.05	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.738	3.001	0.305	1.675	0.718	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	275	0	624	945	0	0	1128
normalized size	1	1.00	0.82	0.00	1.87	2.83	0.00	0.00	3.38
time (sec)	N/A	0.186	0.332	0.294	0.792	0.417	0.000	0.000	1.034

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	231	0	431	679	0	0	641
normalized size	1	1.00	0.84	0.00	1.56	2.46	0.00	0.00	2.32
time (sec)	N/A	0.127	0.292	0.304	0.789	0.431	0.000	0.000	0.829

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	209	0	269	441	0	357	328
normalized size	1	1.00	0.96	0.00	1.23	2.02	0.00	1.64	1.50
time (sec)	N/A	0.099	0.244	0.291	0.920	0.444	0.000	100.515	0.652

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	143	242	632	0	153
normalized size	1	1.00	0.75	0.00	0.89	1.51	3.95	0.00	0.96
time (sec)	N/A	0.069	0.206	0.157	0.574	0.408	75.965	0.000	0.507

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	75	72	187	66	60
normalized size	1	1.00	0.93	1.00	1.23	1.18	3.07	1.08	0.98
time (sec)	N/A	0.015	0.070	0.053	0.618	0.412	17.644	0.204	0.218

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	163	0	186	0	0	0	-1
normalized size	1	1.00	1.10	0.00	1.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.096	0.324	0.929	0.428	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	93	0	123	280	0	190	152
normalized size	1	1.00	0.73	0.00	0.96	2.19	0.00	1.48	1.19
time (sec)	N/A	0.052	0.189	0.309	0.721	109.509	0.000	0.371	1.199

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	206	0	232	0	0	595	384
normalized size	1	1.00	1.02	0.00	1.15	0.00	0.00	2.95	1.90
time (sec)	N/A	0.112	0.530	0.308	0.783	0.000	0.000	0.570	2.709

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	456	0	0	1765	977
normalized size	1	1.00	0.98	0.00	1.75	0.00	0.00	6.79	3.76
time (sec)	N/A	0.152	0.763	0.310	0.765	0.000	0.000	0.851	5.479

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	480	0	776	0	0	3911	2215
normalized size	1	1.00	1.51	0.00	2.44	0.00	0.00	12.30	6.97
time (sec)	N/A	0.195	1.493	0.316	1.034	0.000	0.000	1.173	10.154

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2240	2220	1386	0	1799	0	0	0	-1
normalized size	1	0.99	0.62	0.00	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.436	3.394	0.299	1.308	1.386	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1645	1657	899	0	1123	0	0	0	-1
normalized size	1	1.01	0.55	0.00	0.68	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.758	2.033	0.300	1.181	1.585	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1063	1097	480	0	623	0	0	0	-1
normalized size	1	1.03	0.45	0.00	0.59	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.163	0.928	0.155	1.095	0.872	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	389	0	298	0	0	0	-1
normalized size	1	1.00	1.45	0.00	1.11	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.235	0.114	0.877	1.546	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1471	2096	1370	0	0	0	0	0	-1
normalized size	1	1.42	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.931	0.294	0.323	0.000	0.735	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	832	878	2930	0	745	0	0	0	-1
normalized size	1	1.06	3.52	0.00	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.932	1.344	0.311	1.795	0.617	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1304	1362	15960	0	1857	0	0	0	-1
normalized size	1	1.04	12.24	0.00	1.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.408	6.312	0.326	3.862	1.753	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1957	2013	47127	0	4732	0	0	0	-1
normalized size	1	1.03	24.08	0.00	2.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.102	6.529	0.323	8.912	1.249	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	56	94	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.33	2.24	0.00	-0.02
time (sec)	N/A	0.080	0.032	0.356	0.000	0.785	133.766	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	526	101	65	0	-1
normalized size	1	1.00	1.00	0.00	14.22	2.73	1.76	0.00	-0.03
time (sec)	N/A	0.061	0.009	0.314	1.529	0.950	12.571	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	268	74	65	0	-1
normalized size	1	1.00	1.00	0.00	7.24	2.00	1.76	0.00	-0.03
time (sec)	N/A	0.063	0.011	0.316	1.250	0.833	9.966	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	105	47	61	86	-1
normalized size	1	1.00	1.00	0.00	2.84	1.27	1.65	2.32	-0.03
time (sec)	N/A	0.038	0.007	0.324	1.018	0.885	9.304	0.214	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	36	30	53	31	-1
normalized size	1	1.00	1.00	0.00	1.06	0.88	1.56	0.91	-0.03
time (sec)	N/A	0.068	0.037	0.313	1.758	0.644	23.959	0.241	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	34	29	0	34	-1
normalized size	1	1.00	1.00	0.00	1.00	0.85	0.00	1.00	-0.03
time (sec)	N/A	0.065	0.010	0.308	1.843	0.652	0.000	0.218	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	80	59	0	85	-1
normalized size	1	1.00	1.00	0.00	2.16	1.59	0.00	2.30	-0.03
time (sec)	N/A	0.065	0.011	0.311	2.164	1.099	0.000	0.327	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	83	24	65	58	-1
normalized size	1	1.00	1.00	0.00	2.77	0.80	2.17	1.93	-0.03
time (sec)	N/A	0.023	0.009	0.321	2.531	0.608	6.276	0.276	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	22595	0	0	0	0	0	-1
normalized size	1	1.00	55.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	7.488	0.642	0.000	0.767	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	436	0	0	0	0	0	-1
normalized size	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	1.541	0.512	0.000	0.625	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	166	0	204	0	0	0	-1
normalized size	1	1.00	0.97	0.00	1.19	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.090	0.324	1.197	0.689	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.386	0.519	0.000	0.635	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	2.872	0.528	0.000	0.508	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	1241	0	0	0	0	0	-1
normalized size	1	1.00	3.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.252	1.842	2.920	0.000	0.999	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	839	0	0	0	0	0	-1
normalized size	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	0.883	1.728	0.000	0.643	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	451	0	0	0	0	0	-1
normalized size	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.603	0.433	1.707	0.000	1.144	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	126	0	0	0	-1
normalized size	1	1.00	0.96	0.00	1.56	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.075	0.115	1.140	1.218	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.545	0.487	1.691	0.000	0.669	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.425	2.546	1.725	0.000	0.967	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	5.095	0.423	0.000	1.495	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	5.088	0.319	0.000	0.721	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	66	450	0	0	0	0	-1
normalized size	1	1.00	0.80	5.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	5.033	0.183	0.000	0.635	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	5.100	0.145	0.000	0.589	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	33.658	0.353	0.000	1.094	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	620	649	18164	0	0	0	0	0	-1
normalized size	1	1.05	29.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.133	22.153	12.089	0.000	0.682	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	517	9211	0	0	0	0	0	-1
normalized size	1	1.04	18.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.819	10.211	9.230	0.000	0.717	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	384	1408	0	0	0	0	0	-1
normalized size	1	1.04	3.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	3.166	6.677	0.000	1.269	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.500	2.014	52.396	0.000	0.606	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	1.325	180.000	0.000	1.987	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.519	0.650	0.920	0.000	1.387	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.484	0.639	0.858	0.000	1.713	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	44	662	59	50	44	0	44
normalized size	1	0.00	0.98	14.71	1.31	1.11	0.98	0.00	0.98
time (sec)	N/A	0.516	0.319	1.160	1.022	0.536	2.170	0.000	0.701

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	45	503	58	49	44	0	44
normalized size	1	0.00	1.00	11.18	1.29	1.09	0.98	0.00	0.98
time (sec)	N/A	0.512	0.089	1.059	1.308	1.465	2.146	0.000	0.537

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	461	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	0.386	0.327	0.000	0.715	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	467	0	1047	0	0	0	-1
normalized size	1	1.00	0.85	0.00	1.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	0.255	0.310	2.214	0.559	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	413	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.130	0.313	0.000	1.800	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	468	421	0	349	0	0	0	-1
normalized size	1	1.61	1.45	0.00	1.20	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.109	0.307	1.843	0.584	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	487	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	0.196	0.311	0.000	1.483	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	479	0	969	0	0	0	-1
normalized size	1	1.00	0.80	0.00	1.63	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	0.402	0.314	2.375	0.648	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1046	1046	1240	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.717	1.479	0.946	0.000	0.577	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	831	1105	0	0	0	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.068	4.107	0.828	0.000	0.832	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	539	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	0.730	0.858	0.000	0.710	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	545	515	0	0	0	0	0	-1
normalized size	1	1.36	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	0.353	0.855	0.000	1.474	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	800	625	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.980	0.937	0.832	0.000	0.592	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	995	721	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.291	0.883	0.858	0.000	0.635	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	97	95	0	0	0	-1
normalized size	1	1.00	1.83	2.11	2.07	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.159	0.016	0.076	0.684	0.654	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	141	18	14	0	18
normalized size	1	1.00	1.00	1.45	7.05	0.90	0.70	0.00	0.90
time (sec)	N/A	0.058	0.102	0.045	0.613	0.677	0.291	0.000	0.257

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.014	0.624	0.000	0.946	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	110	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.038	1.093	0.000	0.671	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	559	0	0	0	0	0	-1
normalized size	1	1.00	3.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.454	10.225	0.000	0.710	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	28	44	25	29	20	153	25
normalized size	1	1.00	1.12	1.76	1.00	1.16	0.80	6.12	1.00
time (sec)	N/A	0.012	0.005	0.123	0.631	0.711	0.268	0.180	0.081

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	113	0	0	0	-1
normalized size	1	1.00	0.94	0.00	1.69	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.016	0.620	0.661	0.729	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.026	0.522	0.000	0.642	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	40	28	42	26	29	28
normalized size	1	1.00	1.00	1.43	1.00	1.50	0.93	1.04	1.00
time (sec)	N/A	0.007	0.003	0.058	0.989	0.710	0.277	0.182	0.148

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	118	0	0	0	-1
normalized size	1	1.00	1.58	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.016	0.363	1.050	0.776	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.025	0.302	0.000	0.693	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	79	28	41	26	29	28
normalized size	1	1.00	1.00	2.82	1.00	1.46	0.93	1.04	1.00
time (sec)	N/A	0.007	0.003	0.123	0.893	0.624	0.296	0.252	0.149

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	118	0	0	0	-1
normalized size	1	1.00	1.58	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.014	0.570	1.117	0.799	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	98	0	0	0	0	0	-1
normalized size	1	1.04	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.026	0.558	0.000	0.811	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0	-1
normalized size	1	1.00	0.86	1.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.012	0.050	0.000	0.635	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	68	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.025	1.302	0.000	0.775	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	149	135	879	0	0	0	0	-1
normalized size	1	1.06	0.96	6.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.043	0.049	0.000	0.668	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	96	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.027	2.766	0.000	0.667	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	206	1636	357	0	0	0	0	-1
normalized size	1	1.01	8.02	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.521	0.051	0.000	0.708	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	334	1080	4733	0	0	0	0	-1
normalized size	1	1.04	3.35	14.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	0.283	0.059	0.000	1.342	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	445	908	0	0	0	0	0	-1
normalized size	1	1.03	2.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.815	3.091	0.000	0.604	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [42] had the largest ratio of [.5484]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	20	10	0.95	32	0.312
2	A	16	10	0.92	32	0.312
3	A	12	7	1.00	30	0.233
4	A	14	8	1.00	32	0.250
5	A	18	11	1.09	32	0.344
6	A	22	11	1.06	32	0.344
7	A	4	3	1.00	29	0.103
8	A	4	3	1.00	29	0.103
9	A	4	3	1.00	29	0.103
10	A	4	2	1.00	27	0.074
11	A	6	6	1.00	29	0.207
12	A	5	4	1.00	29	0.138
13	A	4	3	1.00	29	0.103
14	A	4	3	1.00	29	0.103
15	A	4	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	32	14	1.00	31	0.452
17	A	28	14	1.00	31	0.452
18	A	24	14	1.00	31	0.452
19	A	20	13	1.00	29	0.448
20	A	19	15	1.00	31	0.484
21	A	20	12	1.00	31	0.387
22	A	24	13	1.00	31	0.419
23	A	28	13	1.00	31	0.419
24	A	32	13	1.00	31	0.419
25	A	5	2	1.00	29	0.069
26	A	5	2	1.00	29	0.069
27	A	5	2	1.00	29	0.069
28	A	5	2	1.00	27	0.074
29	A	3	3	1.00	21	0.143
30	A	7	4	1.00	29	0.138
31	A	7	3	1.00	29	0.103
32	A	5	2	1.00	29	0.069
33	A	5	2	1.00	29	0.069
34	A	5	2	1.00	29	0.069
35	A	49	14	0.99	31	0.452
36	A	47	15	1.01	31	0.484
37	A	39	15	1.03	29	0.517
38	A	10	9	1.00	23	0.391
39	A	29	14	1.42	31	0.452
40	A	35	12	1.06	31	0.387
41	A	47	16	1.04	31	0.516

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	61	17	1.03	31	0.548
43	A	3	3	1.00	40	0.075
44	A	3	3	1.00	40	0.075
45	A	3	3	1.00	40	0.075
46	A	2	2	1.00	38	0.053
47	A	3	3	1.00	40	0.075
48	A	3	3	1.00	40	0.075
49	A	3	3	1.00	40	0.075
50	A	1	1	1.00	34	0.029
51	A	11	6	1.00	48	0.125
52	A	9	5	1.00	46	0.109
53	A	7	4	1.00	32	0.125
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	13	6	1.00	39	0.154
57	A	11	6	1.00	39	0.154
58	A	9	5	1.00	37	0.135
59	A	5	3	1.00	25	0.120
60	A	0	0	0.00	0	0.000
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	0	0	0.00	0	0.000
64	A	5	5	1.00	26	0.192
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	12	7	1.05	45	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	10	7	1.04	45	0.156
69	A	8	6	1.04	43	0.140
70	A	0	0	0.00	0	0.000
71	A	0	0	0.00	0	0.000
72	A	0	0	0.00	0	0.000
73	A	0	0	0.00	0	0.000
74	F	0	0	N/A	0	N/A
75	F	0	0	N/A	0	N/A
76	A	30	9	1.00	32	0.281
77	A	27	10	1.00	32	0.312
78	A	18	6	1.00	30	0.200
79	A	18	6	1.61	29	0.207
80	A	29	11	1.00	32	0.344
81	A	31	12	1.00	32	0.375
82	A	37	14	1.00	34	0.412
83	A	30	12	1.00	34	0.353
84	A	21	9	1.00	32	0.281
85	A	19	7	1.36	31	0.226
86	A	31	12	1.00	34	0.353
87	A	40	16	1.00	34	0.471
88	A	5	5	1.00	19	0.263
89	A	1	1	1.00	24	0.042
90	A	3	3	1.00	34	0.088
91	A	3	3	1.00	55	0.055
92	A	3	3	1.00	58	0.052

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	4	4	1.00	11	0.364
94	A	5	5	1.00	13	0.385
95	A	7	7	1.00	13	0.538
96	A	2	2	1.00	13	0.154
97	A	6	6	1.00	15	0.400
98	A	4	4	1.00	15	0.267
99	A	2	2	1.00	13	0.154
100	A	6	6	1.00	15	0.400
101	A	4	4	1.04	15	0.267
102	A	1	1	1.00	38	0.026
103	A	2	2	1.00	50	0.040
104	A	3	3	1.06	42	0.071
105	A	2	2	1.00	62	0.032
106	A	3	3	1.01	49	0.061
107	A	7	5	1.04	42	0.119
108	A	8	6	1.03	65	0.092

Chapter 3

Listing of integrals

3.1 $\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=404

$$-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{3fg^2(a+bx)(bc-ad)}{a(c+dx)}$$

```
[Out] -1/2*B*(-a*d+b*c)*g^3*n/a/c/x+A*f^3*x-1/2*B*(b^2/a^2-d^2/c^2)*g^3*n*ln(x)+1
/2*b^2*B*g^3*n*ln(b*x+a)/a^2-3*B*f^2*g*n*ln(x)*ln(1+b*x/a)+B*f^3*(b*x+a)*ln
(e*((b*x+a)/(d*x+c))^n)/b-1/2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/x^2+3*(-a
*d+b*c)*f*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b*x+a
)/(d*x+c))+3*f^2*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^3*n
*ln(d*x+c)/b/d-1/2*B*d^2*g^3*n*ln(d*x+c)/c^2+3*B*f^2*g*n*ln(x)*ln(1+d*x/c)+
3*B*(-a*d+b*c)*f*g^2*n*ln(a-c*(b*x+a)/(d*x+c))/a/c-3*B*f^2*g*n*polylog(2,-b
*x/a)+3*B*f^2*g*n*polylog(2,-d*x/c)
```

Rubi [A] time = 0.49, antiderivative size = 385, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-3Bf^2gn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+3Bf^2gn\text{PolyLog}\left(2,-\frac{dx}{c}\right)-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g^3*n)/(2*a*c*x) + A*f^3*x + (3*B*(b*c - a*d)*f*g^2*n*Log[x
])/(a*c) - (B*(b^2/a^2 - d^2/c^2)*g^3*n*Log[x])/2 - (3*b*B*f*g^2*n*Log[a +
b*x])/a + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1 + (
b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*d*f*g^2*n*Log[c + d*x])/c
 - (B*d^2*g^3*n*Log[c + d*x])/(2*c^2) + 3*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c]
 - 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] + 3*B*f^2*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \int \left(f^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^3}+\frac{3\ldots}{\ldots}\right.$$

$$= f^3\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+\left(3f^2g\right)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{x}$$

$$= Af^3x-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}$$

$$= Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-$$

$$= Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-$$

$$= -\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(bc-ad)(bc}{2}$$

$$= -\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(bc-ad)(bc}{2}$$

Mathematica [A] time = 0.41, size = 336, normalized size = 0.83

$$\frac{Bg^3n\left(\log(x)\left(a^2d^2x-b^2c^2x\right)+b^2c^2x\log(a+bx)+a\left(-ad^2x\log(c+dx)+acd-bc^2\right)\right)}{2a^2c^2x}+3f^2g\log(x)\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^3*x + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*f*g^2*n*((b*c - a*d)*Log[
x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) + (B*g^3*n*((-(b^2*c^2*x)
```

```
+ a^2*d^2*x)*Log[x] + b^2*c^2*x*Log[a + b*x] + a*(-(b*c^2) + a*c*d - a*d^2*
x*Log[c + d*x])))/(2*a^2*c^2*x) - 3*B*f^2*g*n*(Log[x]*(Log[1 + (b*x)/a] - L
og[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLog[2, -((d*x)/c)])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Af^3x^3 + 3\,Af^2gx^2 + 3\,Afg^2x + Ag^3 + \left(Bf^3x^3 + 3\,Bf^2gx^2 + 3\,Bfg^2x + Bg^3\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3*
B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(e*((b*x + a)/(d*x + c))^n))/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(f + \frac{g}{x}\right)^3 \left(B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) + A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int((f+g/x)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$Bf^3n\left(\frac{a\log(bx + a)}{b} - \frac{c\log(dx + c)}{d}\right) - 3\,Bfg^2n\left(\frac{b\log(bx + a)}{a} - \frac{d\log(dx + c)}{c} - \frac{(bc - ad)\log(x)}{ac}\right) + \frac{1}{2}\,Bg^3n\left(\frac{b^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a)
/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(b*
x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b*c - a*d)/(a*c*x) - (b^2*c^2 - a^2*d^
2)*log(x)/(a^2*c^2)) + B*f^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f
^3*x - 3*B*f^2*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/
x, x) + 3*A*f^2*g*log(x) - 3*B*f*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/x - 3*A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/x^2 - 1
/2*A*g^3/x^2
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+\frac{g}{x}\right)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

3.2 $\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=263

$$2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{b}$$

```
[Out] A*f^2*x-2*B*f*g*n*ln(x)*ln(1+b*x/a)+B*f^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)
/b+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b
*x+a)/(d*x+c))+2*f*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^2
*n*ln(d*x+c)/b/d+2*B*f*g*n*ln(x)*ln(1+d*x/c)+B*(-a*d+b*c)*g^2*n*ln(a-c*(b*x
+a)/(d*x+c))/a/c-2*B*f*g*n*polylog(2,-b*x/a)+2*B*f*g*n*polylog(2,-d*x/c)
```

Rubi [A] time = 0.34, antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-2Bfgn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+2Bfgn\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*(b*c - a*d)*g^2*n*Log[x])/(a*c) - (b*B*g^2*n*Log[a + b*x])/a -
 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c +
 d*x))^n])/b - (g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x
]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x
])/(b*d) + (B*d*g^2*n*Log[c + d*x])/c + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] -
 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \int \left(f^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2}+\right.$$

$$= f^2\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+(2fg)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{x}$$

$$= Af^2x-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}+2fg\log(x)\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.$$

$$= Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}$$

$$= Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}$$

$$= Af^2x+\frac{B(bc-ad)g^2n\log(x)}{ac}-\frac{bBg^2n\log(a+bx)}{a}-2Bfgn\log(x$$

$$= Af^2x+\frac{B(bc-ad)g^2n\log(x)}{ac}-\frac{bBg^2n\log(a+bx)}{a}-2Bfgn\log(x$$

Mathematica [A] time = 0.21, size = 217, normalized size = 0.83

$$2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{x}+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^2n(bc-ad)\,\mathrm{l}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*g^2*n*((b*c -
 a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - 2*B*f*g*n*(Log
[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLo
g[2, -((d*x)/c)])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Af^2x^2+2\,Afgx+Ag^2+\left(Bf^2x^2+2\,Bfgx+Bg^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*l
og(e*((b*x + a)/(d*x + c))^n))/x^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\left(f+\frac{g}{x}\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((f+g/x)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$Bf^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)-Bg^2n\left(\frac{b\log(bx+a)}{a}-\frac{d\log(dx+c)}{c}-\frac{(bc-ad)\log(x)}{ac}\right)+Bf^2x\log\left(e\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a -
 d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - log(
(d*x + c)^n) + log(e))/x, x) + 2*A*f*g*log(x) - B*g^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/x - A*g^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+\frac{g}{x}\right)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)\left(fx+g\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**2/x**2,
 x)
```

3.3 $\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=143

$$g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bfn(bc-ad)\log(c+dx)}{bd}-BgnLi_2\left(-\frac{bx}{a}\right)-Bgn$$

```
[Out] A*f*x-B*g*n*ln(x)*ln(1+b*x/a)+B*f*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b+g*ln(
x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f*n*ln(d*x+c)/b/d+B*g*n*ln(
x)*ln(1+d*x/c)-B*g*n*polylog(2,-b*x/a)+B*g*n*polylog(2,-d*x/c)
```

Rubi [A] time = 0.22, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2528, 2486, 31, 2524, 2357, 2317, 2391}

$$-Bgn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+Bgn\text{PolyLog}\left(2,-\frac{dx}{c}\right)+g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c -
 a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyL
og[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \int \left(f\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\right)dx$$

$$= f\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx + g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x}dx$$

$$= Afx + g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + (Bf)\int \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.$$

$$= Afx + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.$$

$$= Afx + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.$$

$$= Afx - Bgn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g\log$$

$$= Afx - Bgn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + g\log$$

<u>Mathematica</u> [A] time = 0.10, size = 135, normalized size = 0.94

$$g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bfn(bc-ad)\log(c+dx)}{bd}-Bgn\left(\log(x)\left(\log\left(\frac{bx}{a}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
B*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Afx + Ag + \left(Bfx + Bg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((A*f*x + A*g + (B*f*x + B*g)*log(e*((b*x + a)/(d*x + c))^n))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(f + \frac{g}{x}\right)\left(B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int((f+g/x)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$Bfn\left(\frac{a\log(bx + a)}{b} - \frac{c\log(dx + c)}{d}\right) + Bfx\log\left(e\left(\frac{bx}{dx + c} + \frac{a}{dx + c}\right)^{n}\right) + Afx - Bg\int -\frac{\log\left((bx + a)^{n}\right) - \log\left((dx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*f*x - B*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^
n) + log(e))/x, x) + A*g*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(f+\frac{g}{x}\right)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(fx+g)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)/x, x)
```

3.4 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$

Optimal. Leaf size=217

$$-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{Bn(bc-ad)\log(c+dx)}{bdf}+\frac{Bgn\text{Li}_2\left(-\frac{b(g+fx)}{af-bg}\right.}{f^2}$$

```
[Out] A*x/f+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/
f+B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^2-g*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(f*x+g)/f^2-B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^2+B*g*n*polylo
g(2,-b*(f*x+g)/(a*f-b*g))/f^2-B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^2
```

Rubi [A] time = 0.33, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2528, 2486, 31, 2524, 2418, 2394, 2393, 2391}

$$\frac{Bgn\text{PolyLog}\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^2}-\frac{Bgn\text{PolyLog}\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^2}-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(\right.}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g +
f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2 - (
B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog[2,
-((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f
 - d*g))])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f(g+fx)}\right)dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f} - \frac{g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}\,dx}{f}$$

$$= \frac{Ax}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx}{f} + \frac{(Bgn)\,\ldots}{\ldots}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{(B(b\ldots}{\ldots}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} - \frac{g\left(A+B\log\left(e\left(\frac{a+\ldots}{c+\ldots}\right.\right.\right.}{f\ldots}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\ldots}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\ldots}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\ldots}{f^2}$$

Mathematica [A] time = 0.15, size = 185, normalized size = 0.85

$$\frac{-g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+Bgn\left(\log(fx+g)\left(\log\left(\frac{f(a+bx)}{af-bg}\right)-\log\left(\frac{f(c+dx)}{cf-dg}\right)\ldots\right.\right.}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*
f*n*Log[c + d*x])/(b*d) - g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g +
f*x] + B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f - d*
g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (
d*(g + f*x))/(-(c*f) + d*g)]))/f^2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+Ax}{fx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="fricas")
```

```
[Out] integral((B*x*log(e*((b*x + a)/(d*x + c))^n) + A*x)/(f*x + g), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{f+\frac{g}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$A\left(\frac{x}{f}-\frac{g\log\left(fx+g\right)}{f^{2}}\right)-B\int -\frac{x\log\left((bx+a)^{n}\right)-x\log\left((dx+c)^{n}\right)+x\log(e)}{fx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="maxima")
```

```
[Out] A*(x/f - g*log(f*x + g)/f^2) - B*integrate(-(x*log((b*x + a)^n) - x*log((d*
x + c)^n) + x*log(e))/(f*x + g), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f+\frac{g}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2} dx$

Optimal. Leaf size=322

$$-\frac{2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3}-\frac{g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}+\frac{Bg^2n(}{f^2(a}$$

```
[Out] A*x/f^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^2-g^2*(b*x+a)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/f^2/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^2+2
*B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^3-2*g*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))*ln(f*x+g)/f^3-2*B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^3+B*(-a*d+b
*c)*g^2*n*ln((f*x+g)/(d*x+c))/f^2/(a*f-b*g)/(c*f-d*g)+2*B*g*n*polylog(2,-b*
(f*x+g)/(a*f-b*g))/f^3-2*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^3
```

Rubi [A] time = 0.49, antiderivative size = 352, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{2Bgn\text{PolyLog}\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^3}-\frac{2Bgn\text{PolyLog}\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^3}-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)}-\frac{2g\log(fx+g)\left(B\log\left(e\left(\right.\right.\right.}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*x)/f^2 - (b*B*g^2*n*Log[a + b*x])/(f^3*(a*f - b*g)) + (B*(a + b*x)*Log[e
*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(f^3*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (B*d*g^
2*n*Log[c + d*x])/(f^3*(c*f - d*g)) + (B*(b*c - a*d)*g^2*n*Log[g + f*x])/(f
^2*(a*f - b*g)*(c*f - d*g)) + (2*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g
 + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^
3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^3 + (2*B*g*n*Po
lyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*B*g*n*PolyLog[2, -((d*(g +
 f*x))/(c*f - d*g))])/f^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \int\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^2}+\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)^2}-\frac{2g\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{f^2(g+fx)}\right.$$

$$=\frac{\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f^2}-\frac{(2g)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}dx}{f^2}+\frac{g^2\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{(g+fx)^2}}{f^2}$$

$$=\frac{Ax}{f^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)}-\frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3}+\frac{B}{\ }$$

$$=\frac{Ax}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)}-\frac{2g\left(A+B\log\left(\right.\right.}{\ }$$

$$=\frac{Ax}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)}-\frac{B(bc-ad)n\log}{bdf^2}$$

$$=\frac{Ax}{f^2}-\frac{bBg^2n\log(a+bx)}{f^3(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{f^3(g+fx)}$$

$$=\frac{Ax}{f^2}-\frac{bBg^2n\log(a+bx)}{f^3(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{f^3(g+fx)}$$

$$=\frac{Ax}{f^2}-\frac{bBg^2n\log(a+bx)}{f^3(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{f^3(g+fx)}$$

Mathematica [A] time = 0.48, size = 295, normalized size = 0.92

$$\frac{-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{fx+g}-2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+\frac{Bg^2n(b\log(a+bx)(dg-cf)+d(af-}{(af-b}}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(g + f*x) - (B*(b*c - a*d)*f*n*Log[c + d*x])/
(b*d) - 2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (B*g^2*n*
(b*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f
*Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((Log[(f*(a + b*x))/(a*
f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g
 + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^3
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax^2}{f^2x^2+2\,fgx+g^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2*log(e*((b*x + a)/(d*x + c))^n) + A*x^2)/(f^2*x^2 + 2*f*g*x
+ g^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-A\left(\frac{g^2}{f^4x+f^3g}-\frac{x}{f^2}+\frac{2\,g\log\left(fx+g\right)}{f^3}\right)-B\int -\frac{x^2\log\left((bx+a)^n\right)-x^2\log\left((dx+c)^n\right)+x^2\log(e)}{f^2x^2+2\,fgx+g^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="maxima")
```

```
[Out] -A*(g^2/(f^4*x + f^3*g) - x/f^2 + 2*g*log(f*x + g)/f^3) - B*integrate(-(x^2
*log((b*x + a)^n) - x^2*log((d*x + c)^n) + x^2*log(e))/(f^2*x^2 + 2*f*g*x +
 g^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{{\left(f+\frac{g}{x}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**2,x)
```

```
[Out] Timed out
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$

Optimal. Leaf size=531

$$\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^4}-\frac{3g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)(af-bg)}-\frac{b^2Bg^{\cdot}}{2f}$$

```
[Out] A*x/f^3+1/2*B*(-a*d+b*c)*g^3*n/f^3/(a*f-b*g)/(c*f-d*g)/(f*x+g)-1/2*b^2*B*g^
3*n*ln(b*x+a)/f^4/(a*f-b*g)^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^3+1/2
*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/f^4/(f*x+g)^2-3*g^2*(b*x+a)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/f^3/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^
3+1/2*B*d^2*g^3*n*ln(d*x+c)/f^4/(c*f-d*g)^2+1/2*B*(-a*d+b*c)*g^3*(a*d*f+b*c
*f-2*b*d*g)*n*ln(f*x+g)/f^3/(a*f-b*g)^2/(c*f-d*g)^2+3*B*g*n*ln(f*(b*x+a)/(a
*f-b*g))*ln(f*x+g)/f^4-3*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(f*x+g)/f^4-3*
B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^4+3*B*(-a*d+b*c)*g^2*n*ln((f*x+g)
/(d*x+c))/f^3/(a*f-b*g)/(c*f-d*g)+3*B*g*n*polylog(2,-b*(f*x+g)/(a*f-b*g))/f
^4-3*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^4
```

Rubi [A] time = 0.80, antiderivative size = 562, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{3BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^4}-\frac{3BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^4}+\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{f^4(fx+g)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x))
 - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) - (3*b*B*g^2*n*Log[a +
b*x])/(f^4*(a*f - b*g)) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f
^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*f^4*(g + f*x)^2) - (3
*g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^4*(g + f*x)) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g
)^2) + (3*B*d*g^2*n*Log[c + d*x])/(f^4*(c*f - d*g)) + (3*B*(b*c - a*d)*g^2*
n*Log[g + f*x])/(f^3*(a*f - b*g)*(c*f - d*g)) + (B*(b*c - a*d)*g^3*(b*c*f +
 a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)^2) + (3*
B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (3*g*(A + B*Log[e*
```

```
((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Log[(f*(c + d*x))/(c
*f - d*g)]*Log[g + f*x])/f^4 + (3*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b
*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^3} - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^3} + \frac{3g^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{f^3(g+fx)^2}\right.$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f^3} - \frac{(3g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}dx}{f^3} + \frac{\left(3g^2\right)\int \frac{A+B\log\left(e\left(\frac{a+}{c+}\right.\right.}{(g+fx)^2}}{f^3}$$

$$= \frac{Ax}{f^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \frac{3g\left(A+B\log\right.}{}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\right.}{f^4(g+}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\right.}{f^4(g+}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+b}{f^4(af-bg)}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+b}{f^4(af-bg)}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+b}{f^4(af-bg)}$$

Mathematica [A] time = 1.22, size = 470, normalized size = 0.89

$$\frac{\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(fx+g)^2} - \frac{6g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{fx+g} - 6g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) + Bg^3n(bc-ad)\left(\frac{\frac{d^2\log(c+dx)}{bc-ad}+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b*(-(
c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*Log[g
 + f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log[a + b
*x])/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (f*((
(a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[g + f*x]
))/(a*f - b*g)^2)/(c*f - d*g)^2) + 6*B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)]
 - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/
(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*f^4)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{Bx^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax^3}{f^3x^3+3\,f^2gx^2+3\,fg^2x+g^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3*log(e*((b*x + a)/(d*x + c))^n) + A*x^3)/(f^3*x^3 + 3*f^2*g*
x^2 + 3*f*g^2*x + g^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(f+g/x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}A\left(\frac{6fg^2x+5g^3}{f^6x^2+2f^5gx+f^4g^2}-\frac{2x}{f^3}+\frac{6g\log\left(fx+g\right)}{f^4}\right)-B\int-\frac{x^3\log\left((bx+a)^n\right)-x^3\log\left((dx+c)^n\right)+x^3\log(e$$
$$f^3x^3+3f^2gx^2+3fg^2x+g^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*g
*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x + c)
^n) + x^3*log(e))/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{{\left(f+\frac{g}{x}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**3,x)
```

```
[Out] Timed out
```

3.7 $\int (a+bx)^4 \log \left(e \left(f (a+bx)^p (c+dx)^q\right)^r\right) \, dx$

Optimal. Leaf size=201

$$\frac{qr(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{qrx(bc-ad)^4}{5d^4}+\frac{qr(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{qr(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{(a+bx)^5\log\left(e\left(f(a+bx\right.\right.}{5b}$$

```
[Out] -1/5*(-a*d+b*c)^4*q*r*x/d^4+1/10*(-a*d+b*c)^3*q*r*(b*x+a)^2/b/d^3-1/15*(-a*
d+b*c)^2*q*r*(b*x+a)^3/b/d^2+1/20*(-a*d+b*c)*q*r*(b*x+a)^4/b/d-1/25*p*r*(b*
x+a)^5/b-1/25*q*r*(b*x+a)^5/b+1/5*(-a*d+b*c)^5*q*r*ln(d*x+c)/b/d^5+1/5*(b*x
+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^4}{5d^4}+\frac{qr(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{qr(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{qr(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{(a+bx)^5\log\left(e\left(f(a+\right.\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*c - a*d)^4*q*r*x)/(5*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
 - ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^
4)/(20*b*d) - (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c -
 a*d)^5*q*r*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{1}{5}(pr)\int (a+bx)^4\, dx$$

$$= -\frac{pr(a+bx)^5}{25b} + \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{(dqr)}{}$$

$$= -\frac{(bc-ad)^4 qrx}{5d^4} + \frac{(bc-ad)^3 qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2 qr(a+bx)^3}{15bd^2}$$

Mathematica [A] time = 0.31, size = 185, normalized size = 0.92

$$\frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(-15b^4(4p+5q)(c+dx)^4(bc-ad)+40b^3(3p+5q)(c+dx)^3(bc-ad)^2-60b^2(2p+5q)(c+dx)^2(bc-ad)^3}{60d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p + 5
*q)*(c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(b*
c - a*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c - a
*d)^5*q*Log[c + d*x]))/d^5 + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/(5*b)
```

fricas [B] time = 0.43, size = 624, normalized size = 3.10

$$-\frac{12\left(b^5d^5p + b^5d^5q\right)rx^5 + 15\left(4\,ab^4d^5p - \left(b^5cd^4 - 5\,ab^4d^5\right)q\right)rx^4 + 20\left(6\,a^2b^3d^5p + \left(b^5c^2d^3 - 5\,ab^4cd^4 + 10\,a^2b^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*p + b^5*d^5*q)*r*x^5 + 15*(4*a*b^4*d^5*p - (b^5*c*d^4 -
 5*a*b^4*d^5)*q)*r*x^4 + 20*(6*a^2*b^3*d^5*p + (b^5*c^2*d^3 - 5*a*b^4*c*d^4
 + 10*a^2*b^3*d^5)*q)*r*x^3 + 30*(4*a^3*b^2*d^5*p - (b^5*c^3*d^2 - 5*a*b^4*
c^2*d^3 + 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*q)*r*x^2 + 60*(a^4*b*d^5*p + (
b^5*c^4*d - 5*a*b^4*c^3*d^2 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4
*b*d^5)*q)*r*x - 60*(b^5*d^5*p*r*x^5 + 5*a*b^4*d^5*p*r*x^4 + 10*a^2*b^3*d^5
*p*r*x^3 + 10*a^3*b^2*d^5*p*r*x^2 + 5*a^4*b*d^5*p*r*x + a^5*d^5*p*r)*log(b*
x + a) - 60*(b^5*d^5*q*r*x^5 + 5*a*b^4*d^5*q*r*x^4 + 10*a^2*b^3*d^5*q*r*x^3
 + 10*a^3*b^2*d^5*q*r*x^2 + 5*a^4*b*d^5*q*r*x + (b^5*c^5 - 5*a*b^4*c^4*d +
10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4)*q*r)*log(d*x + c)
- 60*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x
^2 + 5*a^4*b*d^5*x)*log(e) - 60*(b^5*d^5*r*x^5 + 5*a*b^4*d^5*r*x^4 + 10*a^2
*b^3*d^5*r*x^3 + 10*a^3*b^2*d^5*r*x^2 + 5*a^4*b*d^5*r*x)*log(f))/(b*d^5)
```

giac [B] time = 34.44, size = 560, normalized size = 2.79

$$\frac{a^5pr\log\left(bx+a\right)}{5\,b}-\frac{1}{25}\left(b^4pr+b^4qr-5\,b^4r\log(f)-5\,b^4\right)x^5-\frac{\left(4\,ab^3dpr-b^4cqr+5\,ab^3dqr-20\,ab^3dr\log(f)-20\right.}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/5*a^5*p*r*log(b*x + a)/b - 1/25*(b^4*p*r + b^4*q*r - 5*b^4*r*log(f) - 5*b
^4)*x^5 - 1/20*(4*a*b^3*d*p*r - b^4*c*q*r + 5*a*b^3*d*q*r - 20*a*b^3*d*r*lo
g(f) - 20*a*b^3*d)*x^4/d - 1/15*(6*a^2*b^2*d^2*p*r + b^4*c^2*q*r - 5*a*b^3*
c*d*q*r + 10*a^2*b^2*d^2*q*r - 30*a^2*b^2*d^2*r*log(f) - 30*a^2*b^2*d^2)*x^
3/d^2 + 1/5*(b^4*p*r*x^5 + 5*a*b^3*p*r*x^4 + 10*a^2*b^2*p*r*x^3 + 10*a^3*b*
p*r*x^2 + 5*a^4*p*r*x)*log(b*x + a) + 1/5*(b^4*q*r*x^5 + 5*a*b^3*q*r*x^4 +
10*a^2*b^2*q*r*x^3 + 10*a^3*b*q*r*x^2 + 5*a^4*q*r*x)*log(d*x + c) - 1/10*(4
*a^3*b*d^3*p*r - b^4*c^3*q*r + 5*a*b^3*c^2*d*q*r - 10*a^2*b^2*c*d^2*q*r + 1
0*a^3*b*d^3*q*r - 20*a^3*b*d^3*r*log(f) - 20*a^3*b*d^3)*x^2/d^3 - 1/5*(a^4*
d^4*p*r + b^4*c^4*q*r - 5*a*b^3*c^3*d*q*r + 10*a^2*b^2*c^2*d^2*q*r - 10*a^3
*b*c*d^3*q*r + 5*a^4*d^4*q*r - 5*a^4*d^4*r*log(f) - 5*a^4*d^4)*x/d^4 + 1/5*
(b^4*c^5*q*r - 5*a*b^3*c^4*d*q*r + 10*a^2*b^2*c^3*d^2*q*r - 10*a^3*b*c^2*d^
3*q*r + 5*a^4*c*d^4*q*r)*log(-d*x - c)/d^5
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^4 \ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [B] time = 0.74, size = 395, normalized size = 1.97

$$\frac{1}{5}\left(b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{60\,a^5fp\log(bx+a)}{b} - \frac{12\,b^4d^4f(p+q)x}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*(60*a^5*f*p*log(b*x + a)/b - (12*b^
4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 + 20
*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3 +
30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*a^
2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^3*
d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5*f
*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q + 5*
a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r/f
```

mupad [B] time = 0.70, size = 886, normalized size = 4.41

$$\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(a^4\,x + 2\,a^3\,b\,x^2 + 2\,a^2\,b^2\,x^3 + a\,b^3\,x^4 + \frac{b^4\,x^5}{5}\right) - x^4\left(\frac{b^3\,r\,\left(5\,a\,d\,p + b\,c\,p + 6\,a\,d\,q\right)}{20\,d}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a
*b^3*x^4 + 2*a^2*b^2*x^3) - x^4*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(20*d)
 - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(100*d)) + x^3*((((b^3*r*(5*a*d*p + b*c*
p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*
c))/(15*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/(3*d) + (a*b^3*c*r*(p
+ q))/(15*d)) - x*((a^3*r*(a*d*p + 2*b*c*p + 3*a*d*q))/d - ((5*a*d + 5*b*c)
```

```
*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p
 + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p
 + b*c*p + 3*a*d*q))/d + (a*b^3*c*r*(p + q))/(5*d)))/(5*b*d) - (a*c*((b^3*r
*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d
)))/(b*d) + (2*a^2*b*r*(a*d*p + b*c*p + 2*a*d*q))/d))/(5*b*d) + (a*c*((((b^
3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(2
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d +
(a*b^3*c*r*(p + q))/(5*d)))/(b*d)) - x^2*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d
*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a
*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d + (a*b^3*c*r
*(p + q))/(5*d)))/(10*b*d) - (a*c*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d
) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d)))/(2*b*d) + (a^2*b*r*(a*d*p + b*
c*p + 2*a*d*q))/d) + (log(c + d*x)*((b^4*c^5*q*r)/5 + a^4*c*d^4*q*r + 2*a^2
*b^2*c^3*d^2*q*r - a*b^3*c^4*d*q*r - 2*a^3*b*c^2*d^3*q*r))/d^5 - (b^4*r*x^5
*(p + q))/25 + (a^5*p*r*log(a + b*x))/(5*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.8 $\int (a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=172

$$-\frac{qr(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{qrx(bc-ad)^3}{4d^3}-\frac{qr(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b}+qr(a$$

```
[Out] 1/4*(-a*d+b*c)^3*q*r*x/d^3-1/8*(-a*d+b*c)^2*q*r*(b*x+a)^2/b/d^2+1/12*(-a*d+
b*c)*q*r*(b*x+a)^3/b/d-1/16*p*r*(b*x+a)^4/b-1/16*q*r*(b*x+a)^4/b-1/4*(-a*d+
b*c)^4*q*r*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$\frac{qrx(bc-ad)^3}{4d^3}-\frac{qr(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{qr(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b}+qr(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) +
 ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(
a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x
)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
```

```
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{1}{4}(pr)\int (a+bx)^3\, dx -$$

$$= -\frac{pr(a+bx)^4}{16b} + \frac{(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{(dqr)\int}{}$$

$$= \frac{(bc-ad)^3qrx}{4d^3} - \frac{(bc-ad)^2qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx)^3}{12bd} - $$

Mathematica [A] time = 0.22, size = 154, normalized size = 0.90

$$\frac{\frac{r\left(4b^3(3p+4q)(c+dx)^3(bc-ad)-18b^2(p+2q)(c+dx)^2(bc-ad)^2+12bdx(p+4q)(bc-ad)^3-12q(bc-ad)^4\log(c+dx)-3b^4(p+q)(c+dx)^4\right)}{12d^4} + (a+bx)^4\log\left(e\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c +
 d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c + d*
x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

fricas [B] time = 0.44, size = 469, normalized size = 2.73

$$-\frac{3\left(b^4d^4p + b^4d^4q\right)rx^4 + 4\left(3\,ab^3d^4p - \left(b^4cd^3 - 4\,ab^3d^4\right)q\right)rx^3 + 6\left(3\,a^2b^2d^4p + \left(b^4c^2d^2 - 4\,ab^3cd^3 + 6\,a^2b^2d^4\right)q\right)r}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*p + b^4*d^4*q)*r*x^4 + 4*(3*a*b^3*d^4*p - (b^4*c*d^3 - 4*
a*b^3*d^4)*q)*r*x^3 + 6*(3*a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6
*a^2*b^2*d^4)*q)*r*x^2 + 12*(a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 - 4*a^3*b*d^4)*q)*r*x - 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^4*p*
```

```
r*x^3 + 6*a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b*x +
a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*
a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3)*q*r)*log(d*x + c) - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x)*log(e) - 12*(b^4*d^4*r*x^4 + 4*a*b^3*d^4*r*x^3 + 6*a^2
*b^2*d^4*r*x^2 + 4*a^3*b*d^4*r*x)*log(f))/(b*d^4)
```

giac [B] time = 8.96, size = 407, normalized size = 2.37

$$\frac{a^4 pr\log(bx+a)}{4b} - \frac{1}{16}\left(b^3pr + b^3qr - 4b^3r\log(f) - 4b^3\right)x^4 - \frac{\left(3ab^2dpr - b^3cqr + 4ab^2dqr - 12ab^2dr\log(f) - 12\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/4*a^4*p*r*log(b*x + a)/b - 1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*log(f) - 4*b
^3)*x^4 - 1/12*(3*a*b^2*d*p*r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b^2*d*r*lo
g(f) - 12*a*b^2*d)*x^3/d + 1/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*x^3 + 6*a^2*b*p*r
*x^2 + 4*a^3*p*r*x)*log(b*x + a) + 1/4*(b^3*q*r*x^4 + 4*a*b^2*q*r*x^3 + 6*a
^2*b*q*r*x^2 + 4*a^3*q*r*x)*log(d*x + c) - 1/8*(3*a^2*b*d^2*p*r + b^3*c^2*q
*r - 4*a*b^2*c*d*q*r + 6*a^2*b*d^2*q*r - 12*a^2*b*d^2*r*log(f) - 12*a^2*b*d
^2)*x^2/d^2 - 1/4*(a^3*d^3*p*r - b^3*c^3*q*r + 4*a*b^2*c^2*d*q*r - 6*a^2*b*
c*d^2*q*r + 4*a^3*d^3*q*r - 4*a^3*d^3*r*log(f) - 4*a^3*d^3)*x/d^3 - 1/4*(b^
3*c^4*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2*q*r - 4*a^3*c*d^3*q*r)*log(
-d*x - c)/d^4
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [A] time = 0.91, size = 285, normalized size = 1.66

$$\frac{1}{4}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right) + \frac{\left(\frac{12a^4fp\log(bx+a)}{b} - \frac{3b^3d^3f(p+q)x^4 + 4\left(ab^2d^3f(3p+4\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^4
+ 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p + 2
*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) - b^
3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f*q
 - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c)/
d^4)*r/f
```

mupad [**B**] time = 0.53, size = 501, normalized size = 2.91

$$x^2\left(\frac{\left(\frac{b^2\,r\,\left(4\,a\,d\,p+b\,c\,p+5\,a\,d\,q\right)}{4\,d}-\frac{b^2\,r\,\left(p+q\right)\,\left(4\,a\,d+4\,b\,c\right)}{16\,d}\right)\,\left(4\,a\,d+4\,b\,c\right)}{8\,b\,d}-\frac{a\,b\,r\,\left(3\,a\,d\,p+2\,b\,c\,p+5\,a\,d\,q\right)}{4\,d}+\frac{a\,b^2\,c\,r\,\left(p+q\right)}{8\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^3,x)
```

```
[Out] x^2*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d +
4*b*c))/(16*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*r*(3*a*d*p + 2*b*c*p + 5*a*
d*q))/(4*d) + (a*b^2*c*r*(p + q))/(8*d)) - x^3*((b^2*r*(4*a*d*p + b*c*p + 5
*a*d*q))/(12*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(48*d)) + log(e*(f*(a + b
*x)^p*(c + d*x)^q)^r)*(a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3) -
 x*(((4*a*d + 4*b*c)*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*
(p + q)*(4*a*d + 4*b*c))/(16*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*r*(3*a*d*p
 + 2*b*c*p + 5*a*d*q))/(2*d) + (a*b^2*c*r*(p + q))/(4*d)))/(4*b*d) + (a^2*r
*(2*a*d*p + 3*b*c*p + 5*a*d*q))/(2*d) - (a*c*((b^2*r*(4*a*d*p + b*c*p + 5*a
*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(16*d)))/(b*d)) - (log(c + d
*x)*(b^3*c^4*q*r - 4*a^3*c*d^3*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2*q*
r))/(4*d^4) - (b^3*r*x^4*(p + q))/16 + (a^4*p*r*log(a + b*x))/(4*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.9 $\int (a+bx)^2 \log \left(e \left(f (a+bx)^p (c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=143

$$\frac{qr(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{qrx(bc-ad)^2}{3d^2}+\frac{(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)}{6bd}-\frac{pr(a+$$

```
[Out] -1/3*(-a*d+b*c)^2*q*r*x/d^2+1/6*(-a*d+b*c)*q*r*(b*x+a)^2/b/d-1/9*p*r*(b*x+a
)^3/b-1/9*q*r*(b*x+a)^3/b+1/3*(-a*d+b*c)^3*q*r*ln(d*x+c)/b/d^3+1/3*(b*x+a)^
3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^2}{3d^2}+\frac{qr(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)}{6bd}-\frac{pr(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*c - a*d)^2*q*r*x)/(3*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2)/(6*b*d) - (p
*r*(a + b*x)^3)/(9*b) - (q*r*(a + b*x)^3)/(9*b) + ((b*c - a*d)^3*q*r*Log[c
+ d*x])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
```

```
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{1}{3}(pr)\int (a+bx)^2\,dx -$$

$$= -\frac{pr(a+bx)^3}{9b} + \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{(dqr)\int}{}$$

$$= -\frac{(bc-ad)^2qrx}{3d^2} + \frac{(bc-ad)qr(a+bx)^2}{6bd} - \frac{pr(a+bx)^3}{9b} - \frac{qr(a+bx}{9b}$$

Mathematica [A] time = 0.14, size = 127, normalized size = 0.89

$$\frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(-3b^2(2p+3q)(c+dx)^2(bc-ad)+6bdx(p+3q)(bc-ad)^2-6q(bc-ad)^3\log(c+dx)+2b^3(p+q)(c+dx)^3\right)}{6d^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*(
c + d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))/d
^3 + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

fricas [B] time = 0.43, size = 325, normalized size = 2.27

$$-\frac{2\left(b^3d^3p + b^3d^3q\right)rx^3 + 3\left(2\,ab^2d^3p - \left(b^3cd^2 - 3\,ab^2d^3\right)q\right)rx^2 + 6\left(a^2bd^3p + \left(b^3c^2d - 3\,ab^2cd^2 + 3\,a^2bd^3\right)q\right)rx - 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*p + b^3*d^3*q)*r*x^3 + 3*(2*a*b^2*d^3*p - (b^3*c*d^2 - 3*
a*b^2*d^3)*q)*r*x^2 + 6*(a^2*b*d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b
*d^3)*q)*r*x - 6*(b^3*d^3*p*r*x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*r*x
 + a^3*d^3*p*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3
*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x +
```

```
 c) - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x)*log(e) - 6*(b^3*d^3
*r*x^3 + 3*a*b^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)
```

giac [B] time = 2.36, size = 269, normalized size = 1.88

$$\frac{a^3 pr \log(bx+a)}{3b} - \frac{1}{9}\left(b^2pr + b^2qr - 3b^2r\log(f) - 3b^2\right)x^3 - \frac{\left(2abdpr - b^2cqr + 3abdqr - 6abdr\log(f) - 6abd\right)x}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/3*a^3*p*r*log(b*x + a)/b - 1/9*(b^2*p*r + b^2*q*r - 3*b^2*r*log(f) - 3*b^
2)*x^3 - 1/6*(2*a*b*d*p*r - b^2*c*q*r + 3*a*b*d*q*r - 6*a*b*d*r*log(f) - 6*
a*b*d)*x^2/d + 1/3*(b^2*p*r*x^3 + 3*a*b*p*r*x^2 + 3*a^2*p*r*x)*log(b*x + a)
 + 1/3*(b^2*q*r*x^3 + 3*a*b*q*r*x^2 + 3*a^2*q*r*x)*log(d*x + c) - 1/3*(a^2*
d^2*p*r + b^2*c^2*q*r - 3*a*b*c*d*q*r + 3*a^2*d^2*q*r - 3*a^2*d^2*r*log(f)
- 3*a^2*d^2)*x/d^2 + 1/3*(b^2*c^3*q*r - 3*a*b*c^2*d*q*r + 3*a^2*c*d^2*q*r)*
log(-d*x - c)/d^3
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \ln\left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [A] time = 0.65, size = 194, normalized size = 1.36

$$\frac{1}{3}\left(b^2x^3 + 3abx^2 + 3a^2x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{6a^3fp\log(bx+a)}{b} - \frac{2b^2d^2f(p+q)x^3 + 3\left(abd^2f(2p+3q) - b^2cdfq\right)x^2 + 6}{d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(2
*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*b
*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log(
d*x + c)/d^3)*r/f
```

mupad [B] time = 0.47, size = 255, normalized size = 1.78

$$x\left(\frac{\left(\frac{b\,r\,\left(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q\right)}{3\,d}-\frac{b\,r\,\left(p+q\right)\,\left(3\,a\,d+3\,b\,c\right)}{9\,d}\right)\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}-\frac{a\,r\,\left(a\,d\,p+b\,c\,p+2\,a\,d\,q\right)}{d}+\frac{a\,b\,c\,r\,\left(p+q\right)}{3\,d}\right)-x^2\,\left(\frac{b\,r}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^2,x)
```

```
[Out] x*((((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(3*d) - (b*r*(p + q)*(3*a*d + 3*b*c)
)/(9*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*r*(a*d*p + b*c*p + 2*a*d*q))/d + (a*
b*c*r*(p + q))/(3*d)) - x^2*((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(6*d) - (b*r
*(p + q)*(3*a*d + 3*b*c))/(18*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a
^2*x + (b^2*x^3)/3 + a*b*x^2) + (log(c + d*x)*(b^2*c^3*q*r + 3*a^2*c*d^2*q*
r - 3*a*b*c^2*d*q*r))/(3*d^3) - (b^2*r*x^3*(p + q))/9 + (a^3*p*r*log(a + b*
x))/(3*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.10 $\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$

Optimal. Leaf size=116

$$-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

```
[Out] -1/2*a*p*r*x+1/2*(-a*d+b*c)*q*r*x/d-1/4*b*p*r*x^2-1/4*q*r*(b*x+a)^2/b-1/2*(
-a*d+b*c)^2*q*r*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2495, 43}

$$-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -(a*p*r*x)/2 + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)^2
)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - \frac{1}{2}(pr)\int (a+bx)\,dx - \frac{(a}{}$$

$$= -\frac{1}{2}aprx - \frac{1}{4}bprx^2 + \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - \frac{(dq}{}$$

$$= -\frac{1}{2}aprx + \frac{(bc-ad)qrx}{2d} - \frac{1}{4}bprx^2 - \frac{qr(a+bx)^2}{4b} - \frac{(bc-ad)^2qr\log}{2bd^2}$$

Mathematica [A] time = 0.20, size = 105, normalized size = 0.91

$$\frac{a^2pr\log(a+bx)}{2b} - \frac{dx\left(r(2ad(p+2q)-2bcq+bdx(p+q))-2d(2a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2cqr(bc}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a^2*p*r*Log[a + b*x])/(2*b) - (2*c*(b*c - 2*a*d)*q*r*Log[c + d*x] + d*x*(r
*(-2*b*c*q + 2*a*d*(p + 2*q) + b*d*(p + q)*x) - 2*d*(2*a + b*x)*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(4*d^2)
```

fricas [A] time = 0.42, size = 197, normalized size = 1.70

$$-\frac{\left(b^2d^2p+b^2d^2q\right)rx^2+2\left(abd^2p-\left(b^2cd-2\,abd^2\right)q\right)rx-2\left(b^2d^2prx^2+2\,abd^2prx+a^2d^2pr\right)\log\left(bx+a\right)-2\left(b^2d\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*p + b^2*d^2*q)*r*x^2 + 2*(a*b*d^2*p - (b^2*c*d - 2*a*b*d^2)*
q)*r*x - 2*(b^2*d^2*p*r*x^2 + 2*a*b*d^2*p*r*x + a^2*d^2*p*r)*log(b*x + a) -
 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d*x
+ c) - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x)*log(e) - 2*(b^2*d^2*r*x^2 + 2*a*b*d^2*
r*x)*log(f))/(b*d^2)
```

giac [A] time = 0.71, size = 148, normalized size = 1.28

$$\frac{a^2pr\log\left(bx+a\right)}{2\,b} - \frac{1}{4}\left(bpr+bqr-2\,br\log(f)-2\,b\right)x^2 + \frac{1}{2}\left(bprx^2+2\,aprx\right)\log\left(bx+a\right) + \frac{1}{2}\left(bqrx^2+2\,aqrx\right)\log\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/2*a^2*p*r*log(b*x + a)/b - 1/4*(b*p*r + b*q*r - 2*b*r*log(f) - 2*b)*x^2 +
 1/2*(b*p*r*x^2 + 2*a*p*r*x)*log(b*x + a) + 1/2*(b*q*r*x^2 + 2*a*q*r*x)*log
(d*x + c) - 1/2*(a*d*p*r - b*c*q*r + 2*a*d*q*r - 2*a*d*r*log(f) - 2*a*d)*x/
d - 1/2*(b*c^2*q*r - 2*a*c*d*q*r)*log(-d*x - c)/d^2
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (bx+a)\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [A] time = 0.68, size = 118, normalized size = 1.02

$$\frac{1}{2}\left(bx^{2}+2\,ax\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)+\frac{\left(\frac{2\,a^{2}fp\log(bx+a)}{b}-\frac{bdf\left(p+q\right)x^{2}+2\left(adf\left(p+2\,q\right)-bcfq\right)x}{d}-\frac{2\left(bc^{2}fq-2\,acdfq\right)\log(d}{d^{2}}\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*(2*a^2*f*p*l
og(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d - 2
*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r/f
```

mupad [B] time = 0.40, size = 128, normalized size = 1.10

$$\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)\left(\frac{b\,x^{2}}{2}+a\,x\right)-x\left(\frac{r\,\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{2\,d}-\frac{r\,\left(p+q\right)\,\left(2\,a\,d+2\,b\,c\right)}{4\,d}\right)-\frac{\ln(c+d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a*x + (b*x^2)/2) - x*((r*(2*a*d*p + b
*c*p + 3*a*d*q))/(2*d) - (r*(p + q)*(2*a*d + 2*b*c))/(4*d)) - (log(c + d*x)
*(b*c^2*q*r - 2*a*c*d*q*r))/(2*d^2) - (b*r*x^2*(p + q))/4 + (a^2*p*r*log(a
+ b*x))/(2*b)
```

<u>**sympy**</u> **[A]** time = 60.28, size = 427, normalized size = 3.68

$$\begin{cases} ax\log\left(e\left(a^{p}c^{q}f\right)^{r}\right) \\ a\left(\frac{cqr\log(c+dx)}{d} + prx\log(a) + qrx\log(c+dx) - qrx + rx\log(f) + x\log(e)\right) \\ \frac{a^2pr\log(a+bx)}{2b} + aprx\log(a+bx) - \frac{aprx}{2} + aqrx\log(c) + arx\log(f) + ax\log(e) + \frac{bprx^2\log(a+bx)}{2} - \frac{bprx^2}{4} + \frac{bqrx^2\log}{2} \\ \frac{a^2pr\log(a+bx)}{2b} + \frac{a^2qr\log(c+dx)}{2b} - \frac{a^2qr\log\left(\frac{c}{d}+x\right)}{2b} + \frac{acqr\log\left(\frac{c}{d}+x\right)}{d} + aprx\log(a+bx) - \frac{aprx}{2} + aqrx\log(c+dx) - aqrx + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((a*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a*(c*q*r*log
(c + d*x)/d + p*r*x*log(a) + q*r*x*log(c + d*x) - q*r*x + r*x*log(f) + x*lo
g(e)), Eq(b, 0)), (a**2*p*r*log(a + b*x)/(2*b) + a*p*r*x*log(a + b*x) - a*p
*r*x/2 + a*q*r*x*log(c) + a*r*x*log(f) + a*x*log(e) + b*p*r*x**2*log(a + b*
x)/2 - b*p*r*x**2/4 + b*q*r*x**2*log(c)/2 + b*r*x**2*log(f)/2 + b*x**2*log(
e)/2, Eq(d, 0)), (a**2*p*r*log(a + b*x)/(2*b) + a**2*q*r*log(c + d*x)/(2*b)
 - a**2*q*r*log(c/d + x)/(2*b) + a*c*q*r*log(c/d + x)/d + a*p*r*x*log(a + b
*x) - a*p*r*x/2 + a*q*r*x*log(c + d*x) - a*q*r*x + a*r*x*log(f) + a*x*log(e
) - b*c**2*q*r*log(c/d + x)/(2*d**2) + b*c*q*r*x/(2*d) + b*p*r*x**2*log(a +
 b*x)/2 - b*p*r*x**2/4 + b*q*r*x**2*log(c + d*x)/2 - b*q*r*x**2/4 + b*r*x**
2*log(f)/2 + b*x**2*log(e)/2, True))
```

3.11 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx$

Optimal. Leaf size=107

$$\frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{qr\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

```
[Out] -1/2*p*r*ln(b*x+a)^2/b-q*r*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b+ln(b*x+a)*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-q*r*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b
```

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2494, 2390, 2301, 2394, 2393, 2391}

$$-\frac{qr\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -(p*r*Log[a + b*x]^2)/(2*b) - (q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (q*r*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/b
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx = \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - (pr)\int \frac{\log(a+bx)}{a+bx}\,dx - \frac{(dqr}{}$$

$$= -\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$

$$= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a\right.\right.}{b}$$

$$= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a\right.\right.}{b}$$

Mathematica [A] time = 0.11, size = 93, normalized size = 0.87

$$-\frac{\log(a+bx)\left(-2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+2qr\log\left(\frac{b(c+dx)}{bc-ad}\right)+pr\log(a+bx)\right)+2qr\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -1/2*(Log[a + b*x]*(p*r*Log[a + b*x] + 2*q*r*Log[(b*(c + d*x))/(b*c - a*d)]
 - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*q*r*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/b
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

maxima [A] time = 0.80, size = 164, normalized size = 1.53

$$-\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)fq}{b}-\frac{fp\log(bx+a)^2+2\,fq\log(bx+a)\log(dx+c)}{b}\right)r}{2\,f}-\frac{\left(fp\log\left(bx+a\right)+fq\log\left(dx+c\right)\right)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*f*q/b - (f*p*log(b*x + a)^2 + 2*f*q*log(b*x + a)*log(d*x
 + c))/b)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(b*x + a)/(b*f)
+ log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(b*x + a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a),x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a + b*x), x)
```

3.12 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

```
[Out] -p*r/b/(b*x+a)+d*q*r*ln(b*x+a)/b/(-a*d+b*c)-d*q*r*ln(d*x+c)/b/(-a*d+b*c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2495, 32, 36, 31}

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] -((p*r)/(b*(a + b*x))) + (d*q*r*Log[a + b*x])/(b*(b*c - a*d)) - (d*q*r*Log[
c + d*x])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*
x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (pr)\int \frac{1}{(a+bx)^2}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)(c+dx}$$

$$= -\frac{pr}{b(a+bx)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(dqr)\int \frac{1}{a+bx}\,dx}{bc-ad} - \frac{\left(d^2qr\right)\int}{b(bc-}$$

$$= -\frac{pr}{b(a+bx)} + \frac{dqr\log(a+bx)}{b(bc-ad)} - \frac{dqr\log(c+dx)}{b(bc-ad)} - \frac{\log\left(e\left(f(a+bx)^p(c+\right.\right.}{b(a+bx)}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 0.94

$$\frac{r\left(\frac{dq\log(a+bx)}{bc-ad} - \frac{dq\log(c+dx)}{bc-ad} - \frac{p}{a+bx}\right)}{b} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] (r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(b
*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

fricas [A] time = 0.43, size = 120, normalized size = 1.26

$$-\frac{(bc-ad)pr + (bc-ad)r\log(f) - \left(bdqrx + \left(adq-(bc-ad)p\right)r\right)\log(bx+a) + \left(bdqrx + bcqr\right)\log(dx+c) + (bc}{ab^2c - a^2bd + \left(b^3c - ab^2d\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) - (b*d*q*r*x + (a*d*q - (b*c - a*d
)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c) + (b*c - a*d)*log
(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

giac [A] time = 0.19, size = 112, normalized size = 1.18

$$\frac{dqr\log(bx+a)}{b^2c-abd}-\frac{dqr\log(dx+c)}{b^2c-abd}-\frac{pr\log(bx+a)}{b^2x+ab}-\frac{qr\log(dx+c)}{b^2x+ab}-\frac{pr+r\log(f)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) - p
*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r*l
og(f) + 1)/(b^2*x + a*b)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

maxima [A] time = 0.71, size = 99, normalized size = 1.04

$$\frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)-\frac{bfp}{b^2x+ab}\right)r}{bf}-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*x
 + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)
```

mupad [B] time = 2.16, size = 99, normalized size = 1.04

$$-\frac{\ln\left(e\left(f\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)\,\left(x+\frac{a}{b}\right)}{\left(a+b\,x\right)^2}-\frac{p\,r}{x\,b^2+a\,b}+\frac{d\,q\,r\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^2,x)
```

```
[Out] (d*q*r*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c)) - (p*
r)/(a*b + b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x + a/b))/(a + b*
x)^2
```

sympy [A] time = 139.98, size = 1931, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(2*0**p*zoo**(2*p)*c*q*r*log(c + d*x)/(0**p*zoo**p*d - d) -
0**p*zoo**(2*p)*d*q*r*x/(0**p*zoo**p*d - d) - 3*0**p*zoo**p*c*q*r*log(c + d
*x)/(0**p*zoo**p*d - d) + 0**p*zoo*zoo**p*d*p*r*x/(0**p*zoo**p*d - d) + 0**
p*zoo**p*d*q*r*x*log(c + d*x)/(0**p*zoo**p*d - d) + 0**p*zoo**p*d*q*r*x/(0*
*p*zoo**p*d - d) + 0**p*zoo**p*d*r*x*log(f)/(0**p*zoo**p*d - d) + 0**p*zoo*
*p*d*x*log(e)/(0**p*zoo**p*d - d) + c*q*r*log(c + d*x)/(0**p*zoo**p*d - d)
+ zoo*d*p*r*x/(0**p*zoo**p*d - d) - d*q*r*x*log(c + d*x)/(0**p*zoo**p*d - d
) - d*r*x*log(f)/(0**p*zoo**p*d - d) - d*x*log(e)/(0**p*zoo**p*d - d)), Eq(
a, -b*x)), ((c*q*r*log(c + d*x)/d + p*r*x*log(a) + q*r*x*log(c + d*x) - q*r
*x + r*x*log(f) + x*log(e))/a**2, Eq(b, 0)), (-p*r*log(a + b*x)/(a*b + b**2
*x) - p*r/(a*b + b**2*x) - q*r*log(a*d/b + d*x)/(a*b + b**2*x) - q*r/(a*b +
 b**2*x) - r*log(f)/(a*b + b**2*x) - log(e)/(a*b + b**2*x), Eq(c, a*d/b)),
(-a*d*q*r*log(a/b + x)/(a**2*b*d - a*b**2*c + a*b**2*d*x - b**3*c*x) + b*c*
q*r*log(c + d*x)/(a**2*b*d - a*b**2*c + a*b**2*d*x - b**3*c*x) + b*c*r*log(
f)/(a**2*b*d - a*b**2*c + a*b**2*d*x - b**3*c*x) + b*c*log(e)/(a**2*b*d - a
*b**2*c + a*b**2*d*x - b**3*c*x) + b*d*q*r*x*log(c + d*x)/(a**2*b*d - a*b**
2*c + a*b**2*d*x - b**3*c*x) - b*d*q*r*x*log(a/b + x)/(a**2*b*d - a*b**2*c
+ a*b**2*d*x - b**3*c*x) + b*d*r*x*log(f)/(a**2*b*d - a*b**2*c + a*b**2*d*x
 - b**3*c*x) + b*d*x*log(e)/(a**2*b*d - a*b**2*c + a*b**2*d*x - b**3*c*x),
Eq(p, 0)), (-p*r*log(a + b*x)/(a*b + b**2*x) - p*r/(a*b + b**2*x) - q*r*log
(c)/(a*b + b**2*x) - r*log(f)/(a*b + b**2*x) - log(e)/(a*b + b**2*x), Eq(d,
 0)), (-a*d*p**2*r*log(a + b*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b
**3*c*p*x) - a*d*p**2*r/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*
x) - a*d*p*q*r*log(a + b*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*
c*p*x) - a*d*p*q*r*log(c + d*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b
**3*c*p*x) + a*d*p*q*r*log(c/d + x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x
 - b**3*c*p*x) - a*d*p*r*log(f)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b
**3*c*p*x) - a*d*p*log(e)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*
p*x) - a*d*q**2*r*log(c + d*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b*
*3*c*p*x) + a*d*q**2*r*log(c/d + x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x
 - b**3*c*p*x) - a*d*q*r*log(f)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b
**3*c*p*x) - a*d*q*log(e)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*
p*x) + b*c*p**2*r*log(a + b*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b*
*3*c*p*x) + b*c*p**2*r/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x
```

```
) + b*c*p*q*r*log(c + d*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c
*p*x) + b*c*p*r*log(f)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x
) + b*c*p*log(e)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) - b*
d*p*q*r*x*log(a + b*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x
) + b*d*p*q*r*x*log(c/d + x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3
*c*p*x) - b*d*q**2*r*x*log(c + d*x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x
 - b**3*c*p*x) + b*d*q**2*r*x*log(c/d + x)/(a**2*b*d*p - a*b**2*c*p + a*b**
2*d*p*x - b**3*c*p*x) - b*d*q*r*x*log(f)/(a**2*b*d*p - a*b**2*c*p + a*b**2*
d*p*x - b**3*c*p*x) - b*d*q*x*log(e)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*
x - b**3*c*p*x), True))
```

3.13 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=135

$$-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{dqr}{2b(a+bx)(bc-ad)}-\frac{pr}{4b(a+bx)^2}$$

```
[Out] -1/4*p*r/b/(b*x+a)^2-1/2*d*q*r/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*q*r*ln(b*x+a)/b
/(-a*d+b*c)^2+1/2*d^2*q*r*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)/b/(b*x+a)^2
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{dqr}{2b(a+bx)(bc-ad)}-\frac{pr}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] -(p*r)/(4*b*(a + b*x)^2) - (d*q*r)/(2*b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*L
og[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q*r*Log[c + d*x])/(2*b*(b*c - a*d)^
2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
```

```
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{1}{2}(pr)\int \frac{1}{(a+bx)^3}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)^2}}{2b}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^2} - \right.}{}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{dqr}{2b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr\log(c+d}{2b(bc-ad)^2}$$

Mathematica [A] time = 0.24, size = 116, normalized size = 0.86

$$\frac{r\left(-\frac{d^2q\log(a+bx)}{(bc-ad)^2} + \frac{d^2q\log(c+dx)}{(bc-ad)^2} - \frac{p-\frac{2dq(a+bx)}{ad-bc}}{2(a+bx)^2}\right) - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] (r*(-1/2*(p - (2*d*q*(a + b*x))/(-(b*c) + a*d))/(a + b*x)^2 - (d^2*q*Log[a
+ b*x])/(b*c - a*d)^2 + (d^2*q*Log[c + d*x])/(b*c - a*d)^2) - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2)/(2*b)
```

fricas [B] time = 0.44, size = 323, normalized size = 2.39

$$-\frac{2\left(b^2cd-abd^2\right)qrx + 2\left(b^2c^2-2\,abcd+a^2d^2\right)r\log(f) + \left(\left(b^2c^2-2\,abcd+a^2d^2\right)p + 2\left(abcd-a^2d^2\right)q\right)r + 2\left(b^2\right.}{4\left(a^2b^3c^2-2\,a^3b^2cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*log
(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2*(
b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^2*d^2*q + (b^2*c^2 - 2*a*b*c*d + a^2
```

```
*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2
- 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(e))/
(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3
*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

giac [A] time = 0.22, size = 246, normalized size = 1.82

$$-\frac{d^2qr\log(bx+a)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}+\frac{d^2qr\log(dx+c)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{pr\log(bx+a)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}-\frac{qr\log(dx+c)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}-\frac{2\,bd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*r
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/(b
^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a
^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(f) -
 2*a*d*r*log(f) + 2*b*c - 2*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2
*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{(bx+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

maxima [A] time = 0.75, size = 165, normalized size = 1.22

$$-\frac{\left(2\,dfq\left(\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{abc-a^2d+\left(b^2c-abd\right)x}\right)+\frac{bfp}{b^3x^2+2\,ab^2x+a^2b}\right)r}{4\,bf}-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x +
 c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)/((b*x + a)^2*b)
```

mupad [B] time = 2.39, size = 182, normalized size = 1.35

$$\frac{\frac{b\,c\,p\,r-a\,d\,p\,r+2\,a\,d\,q\,r}{2\,\left(a\,d-b\,c\right)}+\frac{b\,d\,q\,r\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}-\frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)}{{\left(a+b\,x\right)}^3}+\frac{d^2\,q\,r\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^3,x)
```

```
[Out] ((b*c*p*r - a*d*p*r + 2*a*d*q*r)/(2*(a*d - b*c)) + (b*d*q*r*x)/(a*d - b*c))
/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*
(x/2 + a/(2*b)))/(a + b*x)^3 + (d^2*q*r*atanh((2*b^3*c^2 - 2*a^2*b*d^2)/(2*
b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.14 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} \, dx$

Optimal. Leaf size=164

$$\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}+\frac{d^2qr}{3b(a+bx)(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-$$

```
[Out] -1/9*p*r/b/(b*x+a)^3-1/6*d*q*r/b/(-a*d+b*c)/(b*x+a)^2+1/3*d^2*q*r/b/(-a*d+b
*c)^2/(b*x+a)+1/3*d^3*q*r*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d^3*q*r*ln(d*x+c)/b/
(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3
```

Rubi [A] time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$\frac{d^2qr}{3b(a+bx)(bc-ad)^2}+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] -(p*r)/(9*b*(a + b*x)^3) - (d*q*r)/(6*b*(b*c - a*d)*(a + b*x)^2) + (d^2*q*r
)/(3*b*(b*c - a*d)^2*(a + b*x)) + (d^3*q*r*Log[a + b*x])/(3*b*(b*c - a*d)^3
) - (d^3*q*r*Log[c + d*x])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]/(3*b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
```

```
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}+\frac{1}{3}(pr)\int\frac{1}{(a+bx)^4}\,dx+\frac{(dqr)\int\frac{1}{(a+bx)^3}}{3b}$$

$$= -\frac{pr}{9b(a+bx)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}+\frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\right.}{}$$

$$= -\frac{pr}{9b(a+bx)^3}-\frac{dqr}{6b(bc-ad)(a+bx)^2}+\frac{d^2qr}{3b(bc-ad)^2(a+bx)}+\frac{d^3qr\log(}{3b(bc-}$$

Mathematica [A] time = 0.36, size = 141, normalized size = 0.86

$$\frac{r\left(\frac{d^3q\log(a+bx)}{(bc-ad)^3}-\frac{d^3q\log(c+dx)}{(bc-ad)^3}+\frac{\frac{6d^2q(a+bx)^2}{(bc-ad)^2}+\frac{3dq(a+bx)}{ad-bc}-2p}{6(a+bx)^3}\right)-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] (r*((-2*p + (3*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Log[
c + d*x])/(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3
)/(3*b)
```

fricas [B] time = 0.44, size = 580, normalized size = 3.54

$$\frac{6\left(b^3cd^2-ab^2d^3\right)qrx^2-3\left(b^3c^2d-6\,ab^2cd^2+5\,a^2bd^3\right)qrx-6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)r\log(f)-\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c*d^2 - a*b^2*d^3)*q*r*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*
a^2*b*d^3)*q*r*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*r*
log(f) - (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a*b^
2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*q)*r + 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^
3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*p)*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3
*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*
r)*log(d*x + c) - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^
3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a
^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

giac [B] time = 0.23, size = 469, normalized size = 2.86

$$\frac{d^3qr\log(bx+a)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{d^3qr\log(dx+c)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{pr\log(bx+a)}{3\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
 a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b
) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b^2*c^
2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r - 6*b
^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c^2 + 12
*a*b*c*d - 6*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*
a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6
*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

maxima [**A**] time = 0.73, size = 289, normalized size = 1.76

$$\frac{\left(3\left(\frac{2\,d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-bc+3\,ad}{a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\,\left(ab^3c^2-2\,a^2b^2cd+a^3bd\cdots\right.}\right.\right.}{18\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/18*(3*(2*d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 2*d^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) + (2*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*x))*d*f*q - 2*b*f*p/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b))*r/(b
*f) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^3*b)
```

mupad [**B**] time = 2.87, size = 346, normalized size = 2.11

$$\frac{\frac{x\left(5\,a\,b\,d^2\,q\,r-b^2\,c\,d\,q\,r\right)}{2\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,a^2\,d^2\,p\,r+2\,b^2\,c^2\,p\,r-9\,a^2\,d^2\,q\,r-4\,a\,b\,c\,d\,p\,r+3\,a\,b\,c\,d\,q\,r}{6\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b^2\,d^2\,q\,r\,x^2}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}-\frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)\cdots\right.}{(a+b\,x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^4,x)
```

```
[Out] ((x*(5*a*b*d^2*q*r - b^2*c*d*q*r))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (2
*a^2*d^2*p*r + 2*b^2*c^2*p*r - 9*a^2*d^2*q*r - 4*a*b*c*d*p*r + 3*a*b*c*d*q*
r)/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b^2*d^2*q*r*x^2)/(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2) - (log(e
*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/3 + a/(3*b)))/(a + b*x)^4 - (2*d^3*q*r*a
tanh((3*b^4*c^3 + 3*a^3*b*d^3 - 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)/(3*b*(a*d
- b*c)^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(3*b*
(a*d - b*c)^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.15 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx$

Optimal. Leaf size=193

$$-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+\frac{d^4qr\log(c+dx)}{4b(bc-ad)^4}-\frac{d^3qr}{4b(a+bx)(bc-ad)^3}+\frac{d^2qr}{8b(a+bx)^2(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

```
[Out] -1/16*p*r/b/(b*x+a)^4-1/12*d*q*r/b/(-a*d+b*c)/(b*x+a)^3+1/8*d^2*q*r/b/(-a*d
+b*c)^2/(b*x+a)^2-1/4*d^3*q*r/b/(-a*d+b*c)^3/(b*x+a)-1/4*d^4*q*r*ln(b*x+a)/
b/(-a*d+b*c)^4+1/4*d^4*q*r*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r)/b/(b*x+a)^4
```

Rubi [A] time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$-\frac{d^3qr}{4b(a+bx)(bc-ad)^3}+\frac{d^2qr}{8b(a+bx)^2(bc-ad)^2}-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+\frac{d^4qr\log(c+dx)}{4b(bc-ad)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] -(p*r)/(16*b*(a + b*x)^4) - (d*q*r)/(12*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q
*r)/(8*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r)/(4*b*(b*c - a*d)^3*(a + b*x
)) - (d^4*q*r*Log[a + b*x])/(4*b*(b*c - a*d)^4) + (d^4*q*r*Log[c + d*x])/(4
*b*(b*c - a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{1}{4}(pr)\int \frac{1}{(a+bx)^5}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)^4}}{4b}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^4} -\right.}{}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{}{4b(bc}$$

Mathematica [A] time = 0.36, size = 164, normalized size = 0.85

$$\frac{r\left(-\frac{d^4q\log(a+bx)}{(bc-ad)^4}+\frac{d^4q\log(c+dx)}{(bc-ad)^4}+\frac{-\frac{12d^3q(a+bx)^3}{(bc-ad)^3}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}+\frac{4dq(a+bx)}{ad-bc}-3p}{12(a+bx)^4}\right)-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] (r*((-3*p + (4*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q*L
og[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4) - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

fricas [B] time = 0.44, size = 861, normalized size = 4.46

$$-\frac{12\left(b^4cd^3-ab^3d^4\right)qrx^3-6\left(b^4c^2d^2-8ab^3cd^3+7a^2b^2d^4\right)qrx^2+4\left(b^4c^3d-6ab^3c^2d^2+18a^2b^2cd^3-13a^3bd^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
+ 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^
3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2*c^2*d
^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*
q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*r)*log(b
*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2
 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3)*q*r)*log(d*x + c) + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*
a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2
*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^
2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3
*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^
3*c*d^3 + a^7*b^2*d^4)*x)
```

giac [B] time = 0.21, size = 748, normalized size = 3.88

$$-\frac{d^4qr\log(bx+a)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}+\frac{d^4qr\log(dx+c)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}-\frac{}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^
3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*log(b*x +
a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*q*r*
log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
- 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x^2 + 4
*b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3*c^3*p*
r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2*c^2*d*q
*r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*log(f) - 36*a*b^2*c
^2*d*r*log(f) + 36*a^2*b*c*d^2*r*log(f) - 12*a^3*d^3*r*log(f) + 12*b^3*c^3
- 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 12*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*
d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^
6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2
- 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b
^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*
b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{(bx+a)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

maxima [B] time = 0.85, size = 459, normalized size = 2.38

$$-\frac{\left(2\left(\frac{6\,d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{6\,d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(6*d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2 -
7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^
2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
 a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^
5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3
*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x +
 a)^4*b)
```

mupad [B] time = 3.71, size = 526, normalized size = 2.73

$$\frac{\frac{3\,b^3\,c^3\,p\,r-3\,a^3\,d^3\,p\,r+22\,a^3\,d^3\,q\,r-9\,a\,b^2\,c^2\,d\,p\,r+9\,a^2\,b\,c\,d^2\,p\,r+4\,a\,b^2\,c^2\,d\,q\,r-14\,a^2\,b\,c\,d^2\,q\,r}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x\,\left(13\,q\,r\,a^2\,b\,d^3-5\,q\,r\,a\,b^2\,c\,d^2+q\,r\,b^3\,c^2\,d\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x^2}{2\,\left(a^3\,d^3-\right.}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^5,x)
```

```
[Out] ((3*b^3*c^3*p*r - 3*a^3*d^3*p*r + 22*a^3*d^3*q*r - 9*a*b^2*c^2*d*p*r + 9*a^
2*b*c*d^2*p*r + 4*a*b^2*c^2*d*q*r - 14*a^2*b*c*d^2*q*r)/(12*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(13*a^2*b*d^3*q*r + b^3*c^2*d*q*
```

```
r - 5*a*b^2*c*d^2*q*r))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)) + (d*x^2*(7*a*b^2*d^2*q*r - b^3*c*d*q*r))/(2*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b^3*d^3*q*r*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^
3 + 24*a^2*b^3*x^2) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + a/(4*b))
)/(a + b*x)^5 + (d^4*q*r*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 -
 8*a*b^4*c^3*d)/(4*b*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.16 $\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=920

$$-\frac{q^2r^2\log^2(c+dx)(bc-ad)^5}{5bd^5}-\frac{137q^2r^2\log(c+dx)(bc-ad)^5}{150bd^5}-\frac{2pqr^2\log(c+dx)(bc-ad)^5}{25bd^5}-\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{5}$$

```
[Out] -2/5*(-a*d+b*c)^5*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^5+1/5*(b*
x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/25*(-a*d+b*c)^5*p*q*r^2*ln(d*x
+c)/b/d^5-2/5*(-a*d+b*c)^4*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^
4+1/5*(-a*d+b*c)^3*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-2/15
*(-a*d+b*c)^2*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/10*(-a*
d+b*c)*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+2/5*(-a*d+b*c)^5*q
*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^5-2/25*p*r*(b*x+a)^5*ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r)/b-2/25*q*r*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b+2/25*(-a*d+b*c)^4*p*q*r^2*x/d^4+2/5*(-a*d+b*c)^4*q*(p+q)*r^2*x/d^4-7
7/300*(-a*d+b*c)^3*q^2*r^2*(b*x+a)^2/b/d^3+47/450*(-a*d+b*c)^2*q^2*r^2*(b*x
+a)^3/b/d^2-9/200*(-a*d+b*c)*q^2*r^2*(b*x+a)^4/b/d-1/5*a*(-a*d+b*c)^3*p*q*r
^2*x/d^3-1/10*b*(-a*d+b*c)^3*p*q*r^2*x^2/d^3-1/25*(-a*d+b*c)^3*p*q*r^2*(b*x
+a)^2/b/d^3+16/225*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^3/b/d^2-9/200*(-a*d+b*c)*p*
q*r^2*(b*x+a)^4/b/d-2/5*(-a*d+b*c)^5*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c)
)/b/d^5-137/150*(-a*d+b*c)^5*q^2*r^2*ln(d*x+c)/b/d^5-1/5*(-a*d+b*c)^5*q^2*r
^2*ln(d*x+c)^2/b/d^5+2/125*p^2*r^2*(b*x+a)^5/b+2/125*q^2*r^2*(b*x+a)^5/b+77
/150*(-a*d+b*c)^4*q^2*r^2*x/d^4+4/125*p*q*r^2*(b*x+a)^5/b
```

Rubi [A] time = 0.84, antiderivative size = 920, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$-\frac{q^2r^2\log^2(c+dx)(bc-ad)^5}{5bd^5}-\frac{137q^2r^2\log(c+dx)(bc-ad)^5}{150bd^5}-\frac{2pqr^2\log(c+dx)(bc-ad)^5}{25bd^5}-\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -(a*(b*c - a*d)^3*p*q*r^2*x)/(5*d^3) + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
 + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
```

```
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
```

```
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{1}{5}(2pr)\int (a+bx)^4$$

$$= -\frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} + \frac{(a+bx)^5 \log^2\left(\right.}{}$$

$$= \frac{2p^2r^2(a+bx)^5}{125b} - \frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} +$$

$$= \frac{2(bc-ad)^4pqr^2x}{25d^4} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} + \frac{2(bc-ad)^2pqr^2}{75bd^2}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{2(bc-ad)^4q(p+q)r^2}{5d^4}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} +$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} +$$

Mathematica [B] time = 2.87, size = 2508, normalized size = 2.73

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^5*p*q*r^2)/b + (2*a*b^3*c^4*p*q*r^2)/(5*d^4) - (2*a^2*b^2*c^3*p*q*r^2)
/d^3 + (4*a^3*b*c^2*p*q*r^2)/d^2 - (4*a^4*c*p*q*r^2)/d + (2*a^4*p^2*r^2*x)/
25 + (197*a^4*p*q*r^2*x)/150 + (12*b^4*c^4*p*q*r^2*x)/(25*d^4) - (11*a*b^3*
c^3*p*q*r^2*x)/(5*d^3) + (59*a^2*b^2*c^2*p*q*r^2*x)/(15*d^2) - (101*a^3*b*c
*p*q*r^2*x)/(30*d) + 2*a^4*q^2*r^2*x + (137*b^4*c^4*q^2*r^2*x)/(150*d^4) -
(25*a*b^3*c^3*q^2*r^2*x)/(6*d^3) + (22*a^2*b^2*c^2*q^2*r^2*x)/(3*d^2) - (6*
a^3*b*c*q^2*r^2*x)/d + (4*a^3*b*p^2*r^2*x^2)/25 + (283*a^3*b*p*q*r^2*x^2)/3
00 - (7*b^4*c^3*p*q*r^2*x^2)/(50*d^3) + (19*a*b^3*c^2*p*q*r^2*x^2)/(30*d^2)
 - (67*a^2*b^2*c*p*q*r^2*x^2)/(60*d) + a^3*b*q^2*r^2*x^2 - (77*b^4*c^3*q^2*
r^2*x^2)/(300*d^3) + (13*a*b^3*c^2*q^2*r^2*x^2)/(12*d^2) - (5*a^2*b^2*c*q^2
```

```
*r^2*x^2)/(3*d) + (4*a^2*b^2*p^2*r^2*x^3)/25 + (257*a^2*b^2*p*q*r^2*x^3)/45
0 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^3)/(90*d) +
(4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2) - (7*a*b^3*c
*q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p*q*r^2*x^4)/20
0 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 - (9*b^4*c*q^2*r^
2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x^5)/125 + (2*b^4
*q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (2*a^5*p*q*r^2*Log
[c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5) + (2*a*b^3*c^4*p*q
*r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[c + d*x])/(5*d^3) +
 (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4*c*p*q*r^2*Log[c + d*x]
)/(5*d) - (137*b^4*c^5*q^2*r^2*Log[c + d*x])/(150*d^5) + (25*a*b^3*c^4*q^2*
r^2*Log[c + d*x])/(6*d^4) - (22*a^2*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) +
 (6*a^3*b*c^2*q^2*r^2*Log[c + d*x])/d^2 - (2*a^4*c*q^2*r^2*Log[c + d*x])/d
- (b^4*c^5*q^2*r^2*Log[c + d*x]^2)/(5*d^5) + (a*b^3*c^4*q^2*r^2*Log[c + d*x
]^2)/d^4 - (2*a^2*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 + (2*a^3*b*c^2*q^2*r^
2*Log[c + d*x]^2)/d^2 - (a^4*c*q^2*r^2*Log[c + d*x]^2)/d - (2*a^5*p*r*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (2*a^4*p*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/5 - 2*a^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (2*b^4*
c^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^4) + (2*a*b^3*c^3*q*r*
x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^2*b^2*c^2*q*r*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (4*a^3*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r])/d - (4*a^3*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
5 - 2*a^3*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (b^4*c^3*q*r*x^2
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^3) - (a*b^3*c^2*q*r*x^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^2*b^2*c*q*r*x^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/d - (4*a^2*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])/5 - (4*a^2*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - (2*
b^4*c^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*d^2) + (2*a*b^3*c
*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d) - (2*a*b^3*p*r*x^4*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/5 - (a*b^3*q*r*x^4*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/2 + (b^4*c*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(
10*d) - (2*b^4*p*r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 - (2*b^4*q*
r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 + (2*b^4*c^5*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^5) - (2*a*b^3*c^4*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + (4*a^2*b^2*c^3*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^3*b*c^2*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^4*c*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^4*x*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^2 + 2*a^3*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*a^2*b^
2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + (b^4*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/5 +
 (p*r*Log[a + b*x]*(a*d*(a^4*d^4*(288*p - 137*q) - 60*b^4*c^4*q + 270*a*b^3
*c^3*d*q - 470*a^2*b^2*c^2*d^2*q + 385*a^3*b*c*d^3*q)*r - 60*b*c*(b^4*c^4 -
 5*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*q*r*Log[c
 + d*x] + 60*(b*c - a*d)^5*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 60*a^5*d^5*
```

```
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(150*b*d^5) + (2*(b*c - a*d)^5*p*q*r
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*d^5)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b*
x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (bx+a)^4 \ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.10, size = 1421, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/150*(60*a^5*f*p*log(b*x + a)/b - (12*
```

```
b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 +
20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3
+ 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*
a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^
3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5
*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q +
5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f
 - 1/9000*r^2*(60*((12*p*q + 137*q^2)*b^4*c^5*f^2 - 5*(12*p*q + 125*q^2)*a*
b^3*c^4*d*f^2 + 20*(6*p*q + 55*q^2)*a^2*b^2*c^3*d^2*f^2 - 60*(2*p*q + 15*q^
2)*a^3*b*c^2*d^3*f^2 + 60*(p*q + 5*q^2)*a^4*c*d^4*f^2)*log(d*x + c)/d^5 - 3
600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q -
 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*d^4*f^2*p*q - a^5*d^5*f^2*p*q)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))/(b*d^5) - (144*(p^2 + 2*p*q + q^2)*b^5*d^5*f^2*x^5 - 1800*a^5*d^5*f^
2*p^2*log(b*x + a)^2 - 45*(9*(p*q + q^2)*b^5*c*d^4*f^2 - (16*p^2 + 41*p*q +
 25*q^2)*a*b^4*d^5*f^2)*x^4 + 20*((32*p*q + 47*q^2)*b^5*c^2*d^3*f^2 - 5*(29
*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200*q^2)*a^2*b^3*d^5*f
^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5*(38*p*q + 65*q^2)*a*b^
4*c^2*d^3*f^2 + 5*(67*p*q + 100*q^2)*a^2*b^3*c*d^4*f^2 - (48*p^2 + 283*p*q
+ 300*q^2)*a^3*b^2*d^5*f^2)*x^2 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2
*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*
d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) - 1800*(b^5*c^5*f^2*q^2 - 5*a*b^4*c^
4*d*f^2*q^2 + 10*a^2*b^3*c^3*d^2*f^2*q^2 - 10*a^3*b^2*c^2*d^3*f^2*q^2 + 5*a
^4*b*c*d^4*f^2*q^2)*log(d*x + c)^2 + 60*((72*p*q + 137*q^2)*b^5*c^4*d*f^2 -
 5*(66*p*q + 125*q^2)*a*b^4*c^3*d^2*f^2 + 10*(59*p*q + 110*q^2)*a^2*b^3*c^2
*d^3*f^2 - 5*(101*p*q + 180*q^2)*a^3*b^2*c*d^4*f^2 + (12*p^2 + 197*p*q + 30
0*q^2)*a^4*b*d^5*f^2)*x - 60*(60*a*b^4*c^4*d*f^2*p*q - 270*a^2*b^3*c^3*d^2*
f^2*p*q + 470*a^3*b^2*c^2*d^3*f^2*p*q - 385*a^4*b*c*d^4*f^2*p*q + (12*p^2 +
 137*p*q)*a^5*d^5*f^2)*log(b*x + a))/(b*d^5))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(a+b\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

3.17 $\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=805

$$\frac{q^2r^2\log^2(c+dx)(bc-ad)^4}{4bd^4}+\frac{25q^2r^2\log(c+dx)(bc-ad)^4}{24bd^4}+\frac{pqr^2\log(c+dx)(bc-ad)^4}{8bd^4}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c}{2bd^4}$$

```
[Out] 1/4*a*(-a*d+b*c)^2*p*q*r^2*x/d^2-1/8*(-a*d+b*c)^3*p*q*r^2*x/d^3-13/24*(-a*d
+b*c)^3*q^2*r^2*x/d^3-1/2*(-a*d+b*c)^3*q*(p+q)*r^2*x/d^3+1/8*b*(-a*d+b*c)^2
*p*q*r^2*x^2/d^2+1/16*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^2/b/d^2+13/48*(-a*d+b*c)
^2*q^2*r^2*(b*x+a)^2/b/d^2-7/72*(-a*d+b*c)*p*q*r^2*(b*x+a)^3/b/d-7/72*(-a*d
+b*c)*q^2*r^2*(b*x+a)^3/b/d+1/32*p^2*r^2*(b*x+a)^4/b+1/16*p*q*r^2*(b*x+a)^4
/b+1/32*q^2*r^2*(b*x+a)^4/b+1/8*(-a*d+b*c)^4*p*q*r^2*ln(d*x+c)/b/d^4+25/24*
(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)/b/d^4+1/2*(-a*d+b*c)^4*p*q*r^2*ln(-d*(b*x+a)
/(-a*d+b*c))*ln(d*x+c)/b/d^4+1/4*(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)^2/b/d^4+1/2
*(-a*d+b*c)^3*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-1/4*(-a*d+b
*c)^2*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/6*(-a*d+b*c)*q*
r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/8*p*r*(b*x+a)^4*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)/b-1/8*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/
b-1/2*(-a*d+b*c)^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^4+1/4*
(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+1/2*(-a*d+b*c)^4*p*q*r^2*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b/d^4
```

Rubi [A] time = 0.66, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{q^2r^2\log^2(c+dx)(bc-ad)^4}{4bd^4}+\frac{25q^2r^2\log(c+dx)(bc-ad)^4}{24bd^4}+\frac{pqr^2\log(c+dx)(bc-ad)^4}{8bd^4}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
 + (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
```

```
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*d^2) + ((b*c - a*d)*q*r*(a
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])/(2*b*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{(a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{1}{2}(pr)\int (a+bx)^3 \mathrm{lo}$$

$$= -\frac{pr(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} + \frac{(a+bx)^4 \log^2\left(e\right.}{}$$

$$= \frac{p^2r^2(a+bx)^4}{32b} - \frac{pr(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} + \frac{(a}{}$$

$$= -\frac{(bc-ad)^3pqr^2x}{8d^3} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} - \frac{(bc-ad)pqr^2(a}{24bd}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} +$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc}{}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc}{}$$

Mathematica [B] time = 2.09, size = 1853, normalized size = 2.30

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^4*p*q*r^2)/b - (a*b^2*c^3*p*q*r^2)/(2*d^3) + (2*a^2*b*c^2*p*q*r^2)/d^2
 - (3*a^3*c*p*q*r^2)/d + (a^3*p^2*r^2*x)/8 + (37*a^3*p*q*r^2*x)/24 - (5*b^3
*c^3*p*q*r^2*x)/(8*d^3) + (9*a*b^2*c^2*p*q*r^2*x)/(4*d^2) - (35*a^2*b*c*p*q
*r^2*x)/(12*d) + 2*a^3*q^2*r^2*x - (25*b^3*c^3*q^2*r^2*x)/(24*d^3) + (11*a*
b^2*c^2*q^2*r^2*x)/(3*d^2) - (9*a^2*b*c*q^2*r^2*x)/(2*d) + (3*a^2*b*p^2*r^2
*x^2)/16 + (41*a^2*b*p*q*r^2*x^2)/48 + (3*b^3*c^2*p*q*r^2*x^2)/(16*d^2) - (
2*a*b^2*c*p*q*r^2*x^2)/(3*d) + (3*a^2*b*q^2*r^2*x^2)/4 + (13*b^3*c^2*q^2*r^
2*x^2)/(48*d^2) - (5*a*b^2*c*q^2*r^2*x^2)/(6*d) + (a*b^2*p^2*r^2*x^3)/8 + (
25*a*b^2*p*q*r^2*x^3)/72 - (7*b^3*c*p*q*r^2*x^3)/(72*d) + (2*a*b^2*q^2*r^2*
x^3)/9 - (7*b^3*c*q^2*r^2*x^3)/(72*d) + (b^3*p^2*r^2*x^4)/32 + (b^3*p*q*r^2
*x^4)/16 + (b^3*q^2*r^2*x^4)/32 - (a^4*p^2*r^2*Log[a + b*x]^2)/(4*b) + (2*a
^4*p*q*r^2*Log[c + d*x])/b + (b^3*c^4*p*q*r^2*Log[c + d*x])/(8*d^4) - (a*b^
2*c^3*p*q*r^2*Log[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*Log[c + d*x])/(4
*d^2) - (a^3*c*p*q*r^2*Log[c + d*x])/(2*d) + (25*b^3*c^4*q^2*r^2*Log[c + d*
x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) + (9*a^2*b*c^2*q
^2*r^2*Log[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*Log[c + d*x])/d + (b^3*c^4*
q^2*r^2*Log[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 +
(3*a^2*b*c^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2) - (a^3*c*q^2*r^2*Log[c + d*x]^
2)/d - (2*a^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (a^3*p*r*x*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - 2*a^3*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + (b^3*c^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^3)
- (2*a*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (3*a^2*b*c
*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (3*a^2*b*p*r*x^2*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r])/4 - (3*a^2*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/2 - (b^3*c^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*
d^2) + (a*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (a*b^2*p*
r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - (2*a*b^2*q*r*x^3*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/3 + (b^3*c*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(6*d) - (b^3*p*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b
^3*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b^3*c^4*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^4) + (2*a*b^2*c^3*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (3*a^2*b*c^2*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^3*c*q*r*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^3*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + (3*a^2*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/2 + a*b
^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2)/4 + (p*r*Log[a + b*x]*(a*d*(5*a^3*d^3*(9*p - 5*q) + 1
```

```
2*b^3*c^3*q - 42*a*b^2*c^2*d*q + 52*a^2*b*c*d^2*q)*r + 12*b*c*(b^3*c^3 - 4*
a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*q*r*Log[c + d*x] - 12*(b*c - a*d)^
4*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 12*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(24*b*d^4) - ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)])/(2*b*d^4)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a)^3 \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int (bx + a)^3 \ln\left(e\left(f\,(bx + a)^p\,(dx + c)^q\right)^r\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.19, size = 1071, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^
4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p +
 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) -
b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f
*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c
)/d^4)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*((3*p*q + 2
5*q^2)*b^3*c^4*f^2 - 4*(3*p*q + 22*q^2)*a*b^2*c^3*d*f^2 + 18*(p*q + 6*q^2)*
a^2*b*c^2*d^2*f^2 - 12*(p*q + 4*q^2)*a^3*c*d^3*f^2)*log(d*x + c)/d^4 - 144*
(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^
3*b*c*d^3*f^2*p*q + a^4*d^4*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
 a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^4) + (9*(p^2 + 2*p*q +
 q^2)*b^4*d^4*f^2*x^4 - 72*a^4*d^4*f^2*p^2*log(b*x + a)^2 - 4*(7*(p*q + q^2
)*b^4*c*d^3*f^2 - (9*p^2 + 25*p*q + 16*q^2)*a*b^3*d^4*f^2)*x^3 + 6*((9*p*q
+ 13*q^2)*b^4*c^2*d^2*f^2 - 8*(4*p*q + 5*q^2)*a*b^3*c*d^3*f^2 + (9*p^2 + 41
*p*q + 36*q^2)*a^2*b^2*d^4*f^2)*x^2 + 144*(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*
f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^3*b*c*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) + 72*(b^4*c^4*f^2*q^2 - 4*a*b^3*c^3*d*f^2*q^2 + 6*a^2*b^2*c^2*d
^2*f^2*q^2 - 4*a^3*b*c*d^3*f^2*q^2)*log(d*x + c)^2 - 12*(5*(3*p*q + 5*q^2)*
b^4*c^3*d*f^2 - 2*(27*p*q + 44*q^2)*a*b^3*c^2*d^2*f^2 + 2*(35*p*q + 54*q^2)
*a^2*b^2*c*d^3*f^2 - (3*p^2 + 37*p*q + 48*q^2)*a^3*b*d^4*f^2)*x + 12*(12*a*
b^3*c^3*d*f^2*p*q - 42*a^2*b^2*c^2*d^2*f^2*p*q + 52*a^3*b*c*d^3*f^2*p*q - (
3*p^2 + 25*p*q)*a^4*d^4*f^2)*log(b*x + a))/(b*d^4))/f^2
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

3.18 $\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=686

$$\frac{2qr(bc-ad)^3\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3}-\frac{2pqr^2(bc-ad)^3\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}-\frac{2pqr^2(bc-ad)^3\log(c+}{9bd^3}$$

```
[Out] -1/3*a*(-a*d+b*c)*p*q*r^2*x/d+2/9*(-a*d+b*c)^2*p*q*r^2*x/d^2+5/9*(-a*d+b*c)
^2*q^2*r^2*x/d^2+2/3*(-a*d+b*c)^2*q*(p+q)*r^2*x/d^2-1/6*b*(-a*d+b*c)*p*q*r^
2*x^2/d-1/9*(-a*d+b*c)*p*q*r^2*(b*x+a)^2/b/d-5/18*(-a*d+b*c)*q^2*r^2*(b*x+a
)^2/b/d+2/27*p^2*r^2*(b*x+a)^3/b+4/27*p*q*r^2*(b*x+a)^3/b+2/27*q^2*r^2*(b*x
+a)^3/b-2/9*(-a*d+b*c)^3*p*q*r^2*ln(d*x+c)/b/d^3-11/9*(-a*d+b*c)^3*q^2*r^2*
ln(d*x+c)/b/d^3-2/3*(-a*d+b*c)^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c
)/b/d^3-1/3*(-a*d+b*c)^3*q^2*r^2*ln(d*x+c)^2/b/d^3-2/3*(-a*d+b*c)^2*q*r*(b*
x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/3*(-a*d+b*c)*q*r*(b*x+a)^2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-2/9*p*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)/b-2/9*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2/3*(-a*d+b*c
)^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3+1/3*(b*x+a)^3*ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/3*(-a*d+b*c)^3*p*q*r^2*polylog(2,b*(d*x+c)
/(-a*d+b*c))/b/d^3
```

Rubi [A] time = 0.53, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$-\frac{2pqr^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{2qr(bc-ad)^3\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3}-\frac{2qr(a+bx)(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -(a*(b*c - a*d)*p*q*r^2*x)/(3*d) + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5
*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^
2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/
(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*
x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27
*b) - (2*(b*c - a*d)^3*p*q*r^2*Log[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*
q^2*r^2*Log[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*Log[-((d*(a + b*
x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*Log[c +
d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p
```

```
*(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(9*b) + (2*(b*c - a*d)^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b)
 - (2*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
```

```
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{1}{3}(2pr)\int (a+bx)^2 \mathrm{lo}$$

$$= -\frac{2pr(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} + \frac{(a+bx)^3 \log^2\left(e\right.}{}$$

$$= \frac{2p^2r^2(a+bx)^3}{27b} - \frac{2pr(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} + $$

$$= \frac{2(bc-ad)^2pqr^2x}{9d^2} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} + \frac{2p^2r^2(a+bx)^3}{27b} + $$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{2(bc-ad)^2q(p+q)r^2x}{3d^2}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2} + \frac{2(b}{}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2} + \frac{2(b}{}$$

Mathematica [A] time = 1.29, size = 1211, normalized size = 1.77

$$\frac{1}{54}\left(\frac{108pqr^2a^3}{b}-\frac{18p^2r^2\log^2(a+bx)a^3}{b}+\frac{108pqr^2\log(c+dx)a^3}{b}-\frac{108pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)a^3}{b}-\frac{10\cdots}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] ((108*a^3*p*q*r^2)/b + (36*a*b*c^2*p*q*r^2)/d^2 - (108*a^2*c*p*q*r^2)/d + 1
2*a^2*p^2*r^2*x + 102*a^2*p*q*r^2*x + (48*b^2*c^2*p*q*r^2*x)/d^2 - (126*a*b
*c*p*q*r^2*x)/d + 108*a^2*q^2*r^2*x + (66*b^2*c^2*q^2*r^2*x)/d^2 - (162*a*b
*c*q^2*r^2*x)/d + 12*a*b*p^2*r^2*x^2 + 39*a*b*p*q*r^2*x^2 - (15*b^2*c*p*q*r
^2*x^2)/d + 27*a*b*q^2*r^2*x^2 - (15*b^2*c*q^2*r^2*x^2)/d + 4*b^2*p^2*r^2*x
^3 + 8*b^2*p*q*r^2*x^3 + 4*b^2*q^2*r^2*x^3 - (18*a^3*p^2*r^2*Log[a + b*x]^2
)/b + (108*a^3*p*q*r^2*Log[c + d*x])/b - (12*b^2*c^3*p*q*r^2*Log[c + d*x])/
d^3 + (36*a*b*c^2*p*q*r^2*Log[c + d*x])/d^2 - (36*a^2*c*p*q*r^2*Log[c + d*x
])/d - (66*b^2*c^3*q^2*r^2*Log[c + d*x])/d^3 + (162*a*b*c^2*q^2*r^2*Log[c +
 d*x])/d^2 - (108*a^2*c*q^2*r^2*Log[c + d*x])/d - (18*b^2*c^3*q^2*r^2*Log[c
 + d*x]^2)/d^3 + (54*a*b*c^2*q^2*r^2*Log[c + d*x]^2)/d^2 - (54*a^2*c*q^2*r^
2*Log[c + d*x]^2)/d - (108*a^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b
- 36*a^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a^2*q*r*x*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - (36*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/d^2 + (108*a*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d
 - 36*a*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 54*a*b*q*r*x^2*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (18*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/d - 12*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] -
12*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (36*b^2*c^3*q*r*Log[c
 + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (108*a*b*c^2*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (108*a^2*c*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + 54*a^2*x*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^2 + 54*a*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 18
*b^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (6*p*r*Log[a + b*x]*(a*d*
(a^2*d^2*(16*p - 11*q) - 6*b^2*c^2*q + 15*a*b*c*d*q)*r - 6*b*c*(b^2*c^2 - 3
*a*b*c*d + 3*a^2*d^2)*q*r*Log[c + d*x] + 6*(b*c - a*d)^3*q*r*Log[(b*(c + d*
x))/(b*c - a*d)] + 6*a^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b*d^3)
 + (36*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d
^3))/54
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^2+2\,abx+a^2\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \log\left(\left(\left((bx+a)^p (dx+c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \ln\left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 0.88, size = 769, normalized size = 1.12

$$\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2+\frac{\left(\frac{6\,a^3fp\log(bx+a)}{b}-\frac{2\,b^2d^2f(p+q)x^3+3\left(abd^2f(2\,p+3\,q)-b^2cdfq\right)x^2+6}{d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(
2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*
b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log
(d*x + c)/d^3)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*((2*p
*q + 11*q^2)*b^2*c^3*f^2 - 3*(2*p*q + 9*q^2)*a*b*c^2*d*f^2 + 6*(p*q + 3*q^2
)*a^2*c*d^2*f^2)*log(d*x + c)/d^3 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2
*p*q + 3*a^2*b*c*d^2*f^2*p*q - a^3*d^3*f^2*p*q)*(log(b*x + a)*log((b*d*x +
```

```
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^3) - (4*(p^
2 + 2*p*q + q^2)*b^3*d^3*f^2*x^3 - 18*a^3*d^3*f^2*p^2*log(b*x + a)^2 - 3*(5
*(p*q + q^2)*b^3*c*d^2*f^2 - (4*p^2 + 13*p*q + 9*q^2)*a*b^2*d^3*f^2)*x^2 -
36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2*p*q)*log(b*
x + a)*log(d*x + c) - 18*(b^3*c^3*f^2*q^2 - 3*a*b^2*c^2*d*f^2*q^2 + 3*a^2*b
*c*d^2*f^2*q^2)*log(d*x + c)^2 + 6*((8*p*q + 11*q^2)*b^3*c^2*d*f^2 - 3*(7*p
*q + 9*q^2)*a*b^2*c*d^2*f^2 + (2*p^2 + 17*p*q + 18*q^2)*a^2*b*d^3*f^2)*x -
6*(6*a*b^2*c^2*d*f^2*p*q - 15*a^2*b*c*d^2*f^2*p*q + (2*p^2 + 11*p*q)*a^3*d^
3*f^2)*log(b*x + a))/(b*d^3))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral((a + b*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.19 $\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$

Optimal. Leaf size=540

$$-\frac{qr(bc-ad)^2\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)}{2bd^2}$$

```
[Out] 1/2*a*p^2*r^2*x+1/2*a*p*q*r^2*x-1/2*(-a*d+b*c)*p*q*r^2*x/d-1/2*(-a*d+b*c)*q
^2*r^2*x/d-(-a*d+b*c)*q*(p+q)*r^2*x/d+1/4*b*p^2*r^2*x^2+1/4*b*p*q*r^2*x^2+1
/4*p*q*r^2*(b*x+a)^2/b+1/4*q^2*r^2*(b*x+a)^2/b+1/2*(-a*d+b*c)^2*p*q*r^2*ln(
d*x+c)/b/d^2+3/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c)/b/d^2+(-a*d+b*c)^2*p*q*r^2*
ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^2+1/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c
)^2/b/d^2+(-a*d+b*c)*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/2*p*
r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/2*q*r*(b*x+a)^2*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b-(-a*d+b*c)^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+(-a*d+b*c)^2
*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^2
```

Rubi [A] time = 0.39, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2498, 2495, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{pqr^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{qr(bc-ad)^2\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*p^2*r^2*x)/2 + (a*p*q*r^2*x)/2 - ((b*c - a*d)*p*q*r^2*x)/(2*d) - ((b*c -
 a*d)*q^2*r^2*x)/(2*d) - ((b*c - a*d)*q*(p + q)*r^2*x)/d + (b*p^2*r^2*x^2)/
4 + (b*p*q*r^2*x^2)/4 + (p*q*r^2*(a + b*x)^2)/(4*b) + (q^2*r^2*(a + b*x)^2)
/(4*b) + ((b*c - a*d)^2*p*q*r^2*Log[c + d*x])/(2*b*d^2) + (3*(b*c - a*d)^2*
q^2*r^2*Log[c + d*x])/(2*b*d^2) + ((b*c - a*d)^2*p*q*r^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + ((b*c - a*d)^2*q^2*r^2*Log[c + d*x]
^2)/(2*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(b*d) - (p*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
- (q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b) - ((b*c - a*
d)^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*d^2) + ((a +
 b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) + ((b*c - a*d)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - (pr)\int (a+bx)\log\Big($$

$$= -\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} + \frac{(a+bx)^2\log^2\Big(e\Big(}{}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{4}bp^2r^2x^2 - \frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right.}{2b}$$

$$= \frac{1}{2}ap^2r^2x - \frac{(bc-ad)pqr^2x}{2d} + \frac{1}{4}bp^2r^2x^2 + \frac{pqr^2(a+bx)^2}{4b} + \frac{(bc-}{}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q(p+q)r^2x}{d} +$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - \frac{(bc-}{}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - \frac{(bc-}{}$$

Mathematica [A] time = 0.68, size = 781, normalized size = 1.45

$$\frac{-8a^2d^2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2a^2d^2p^2r^2\log^2(a+bx) + 8a^2d^2pqr^2\log(c+dx) + 8a^2d^2pqr^2 - 4b^2c^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-4*a*b*c*d*p*q*r^2 + 8*a^2*d^2*p*q*r^2 + 2*a*b*d^2*p^2*r^2*x - 6*b^2*c*d*p
*q*r^2*x + 10*a*b*d^2*p*q*r^2*x - 6*b^2*c*d*q^2*r^2*x + 8*a*b*d^2*q^2*r^2*x
 + b^2*d^2*p^2*r^2*x^2 + 2*b^2*d^2*p*q*r^2*x^2 + b^2*d^2*q^2*r^2*x^2 - 2*a^
2*d^2*p^2*r^2*Log[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*Log[c + d*x] - 4*a*b*c*d*p
*q*r^2*Log[c + d*x] + 8*a^2*d^2*p*q*r^2*Log[c + d*x] + 6*b^2*c^2*q^2*r^2*Lo
g[c + d*x] - 8*a*b*c*d*q^2*r^2*Log[c + d*x] + 2*b^2*c^2*q^2*r^2*Log[c + d*x
]^2 - 4*a*b*c*d*q^2*r^2*Log[c + d*x]^2 - 8*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p
```

```
*(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4
*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*
b^2*c^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 8*a*b*c*d*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d^2*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*Log[c + d*x] - 2*(
b*c - a*d)^2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*d*(p - q)*r + 2*
b*c*q*r + 2*a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4*(b*c - a*d)^2*p*
q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*d^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((bx+a)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (bx+a)\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.13, size = 504, normalized size = 0.93

$$\frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 + \frac{\left(\frac{2\,a^2fp\log(bx+a)}{b} - \frac{bdf\left(p+q\right)x^2+2\left(adf\left(p+2\,q\right)-bcfq\right)x}{d} - \frac{2\left(bc^2fq-2\,acdfq\right)\log(dx}{d^2}\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*(2*a^2*f*p
*log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d -
 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r*log(((b*x + a)^p*(d*x + c)
^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*c*d*
f^2)*log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d^2*f^
2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a
*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*log(b*x + a)^2 - (p^2 + 2*p*
q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*log(b*x
+ a)*log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*log(d*x + c)^2
+ 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^2)*x - 2*(
2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*log(b*x + a))/(b*d^2))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)^2\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral((a + b*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.20 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx$

Optimal. Leaf size=431

$$-\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left(\right.\right.}{bpr}\right.$$

```
[Out] 1/3*ln((b*x+a)^(p*r))^3/b/p/r-q*ln((b*x+a)^(p*r))^2*ln(b*(d*x+c)/(-a*d+b*c)
)/b/p+ln((b*x+a)^(p*r))^2*ln((d*x+c)^(q*r))/b/p/r+ln(-d*(b*x+a)/(-a*d+b*c))
*ln((d*x+c)^(q*r))^2/b-2*q*r*ln((b*x+a)^(p*r))*polylog(2,-d*(b*x+a)/(-a*d+b
*c))/b+2*q*r*ln((d*x+c)^(q*r))*polylog(2,b*(d*x+c)/(-a*d+b*c))/b-1/4*(ln((b
*x+a)^(p*r))+ln((d*x+c)^(q*r))-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*((ln((b*x+a
)^(p*r))-ln((d*x+c)^(q*r))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))^2/b/p/r+8*ln(-d
*(b*x+a)/(-a*d+b*c))*ln((d*x+c)^(q*r))/b+8*q*r*polylog(2,b*(d*x+c)/(-a*d+b*
c))/b)+2*p*q*r^2*polylog(3,-d*(b*x+a)/(-a*d+b*c))/b-2*q^2*r^2*polylog(3,b*(
d*x+c)/(-a*d+b*c))/b
```

Rubi [A] time = 0.49, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {2496, 6742, 2390, 2302, 30, 2433, 2375, 2317, 2374, 6589, 2396, 2394, 2393, 2391, 6686}

$$-\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(8\left(\frac{qr\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log\left(-\frac{d(a\right.}{bc\right.}}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d*
x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(b*
p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2*q*
r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r
*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - ((Log[(a +
 b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r])^2/(b*p*r) + 8*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)
^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)))/4 + (2*p*q*r^
2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[3, (b*(c
 + d*x))/(b*c - a*d)])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2496

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Int[(Log[(a + b*x)^(p*r)] + Lo
g[(c + d*x)^(q*r)])^2/(g + h*x), x] + Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] - Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])*(2*Int[Log[(c + d*x)^(
q*r)]/(g + h*x), x] + Int[(Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(g + h*x), x]), x] /; FreeQ[{a, b, c, d
, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[b*g - a*h, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx = \int \frac{\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)^2}{a+bx}\,dx - \Big(\log\left((a+bx)^{pr}\right)+\log\left((c+\right.$$

$$= -\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\Bigg(\Big(\mathrm{l}$$

$$= 2\int \frac{\log\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{a+bx}\,dx - \Big(\log\left((a+bx)^{pr}\right)+\log\left((c+dx\right.$$

$$= \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - \Big(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\mathrm{lo}$$

$$= \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - \Big(\mathrm{log}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2\left((a+bx)^{pr}\right)\mathrm{log}}{bpr}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2\left((a+bx)^{pr}\right)\mathrm{log}}{bpr}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2\left((a+bx)^{pr}\right)\mathrm{log}}{bpr}$$

Mathematica [A] time = 0.20, size = 460, normalized size = 1.07

$$\frac{6qr\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(a+bx)\right)-3pr\log^2(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] (p^2*r^2*Log[a + b*x]^3 + 6*p*q*r^2*Log[a + b*x]^2*Log[c + d*x] - 6*p*q*r^2
*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*q^2*r^2*Lo
g[a + b*x]*Log[c + d*x]^2 - 3*q^2*r^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x]^2 - 3*p*q*r^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*p
*r*Log[a + b*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 6*q*r*Log[a + b*x]
*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 6*q*r*Log[(d*(a + b*x)
)/(-(b*c) + a*d)]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 3*Log
[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*p*q*r^2*Log[a + b*x]*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*q*r*(-(p*r*Log[a + b*x]) + Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 6
*p*q*r^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*q^2*r^2*PolyLog[3, (b
*(c + d*x))/(b*c - a*d)])/(3*b)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)^{2}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(bx+a\right)\log\left(\left((dx+c)^{q}\right)^{r}\right)^{2}}{b}+\int \frac{\left(r^{2}\log(f)^{2}+2\,r\log(e)\log(f)+\log(e)^{2}\right)bdx+\left(r^{2}\log(f)^{2}+2\,r\log(e)\log(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((r^2*log(f)^2 + 2*r*log(
e)*log(f) + log(e)^2)*b*d*x + (r^2*log(f)^2 + 2*r*log(e)*log(f) + log(e)^2)
*b*c + (b*d*x + b*c)*log(((b*x + a)^p)^r)^2 + 2*((r*log(f) + log(e))*b*d*x
+ (r*log(f) + log(e))*b*c)*log(((b*x + a)^p)^r) + 2*((r*log(f) + log(e))*b*
d*x + (r*log(f) + log(e))*b*c - (b*d*q*r*x + a*d*q*r)*log(b*x + a) + (b*d*x
 + b*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^r))/(b^2*d*x^2 + a*b*c + (b
^2*c + a*b*d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)^{2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^{p}\,(c+dx)^{q}\right)^{r}\right)^{2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x), x)
```

3.21 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=465

$$-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{2dqr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)}-\frac{2dqr\log(c+dx)\log\left(e\left(f(a+\right.\right.}{b(bc-ad)}$$

```
[Out] -2*p^2*r^2/b/(b*x+a)+2*d*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)-d*p*q*r^2*ln(b*x+a)
^2/b/(-a*d+b*c)-2*d*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)+2*d*p*q*r^2*ln(-d*(b*x+a
)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)+d*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)-2*d
*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)-2*p*r*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)/b/(b*x+a)+2*d*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)-2*d*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d
+b*c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)-2*d*q^2*r^2*polylog(2,-d*
(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)+2*d*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c)
)/b/(-a*d+b*c)
```

Rubi [A] time = 0.39, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2498, 2495, 32, 36, 31, 2514, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2dpqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}-\frac{2dq^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{2dqr\log(a+bx)\log}{b(}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*Log[a + b*x])/(b*(b*c - a*d)) - (
d*p*q*r^2*Log[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*Log[c + d*x])/(b*(
b*c - a*d)) + (2*d*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(b*(b*c - a*d)) + (d*q^2*r^2*Log[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(a + b*x)) + (2*d*q*r*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - (2*d*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(b*(a + b*x)) - (2*d*q^2*r^2*PolyLog[2, -((d*(a + b*x))
/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{(a+bx)^2}$$

$$= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} +$$

$$= -\frac{2p^2r^2}{b(a+bx)} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\log^2\left(e\left(f(a+bx)^p(c\right.\right.}{b(a+bx)}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} - \frac{2pr\log\left(e\left(f(\right.\right.}{b}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\right.}{b(}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c}{b(bc-ad}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c}{b(bc-ad}$$

Mathematica [A] time = 0.71, size = 411, normalized size = 0.88

$$\frac{-bc\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + ad\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2bcpr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*b*c*p^2*r^2 + 2*a*d*p^2*r^2 - d*p*q*r^2*(a + b*x)*Log[a + b*x]^2 - 2*a*
d*p*q*r^2*Log[c + d*x] - 2*b*d*p*q*r^2*x*Log[c + d*x] + a*d*q^2*r^2*Log[c +
 d*x]^2 + b*d*q^2*r^2*x*Log[c + d*x]^2 - 2*b*c*p*r*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r] + 2*a*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*c*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^2 + a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*d*q*r*(a + b*x)*Lo
```

```
g[a + b*x]*(p*r + p*r*Log[c + d*x] - (p + q)*r*Log[(b*(c + d*x))/(b*c - a*d
)] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) - 2*d*q*(p + q)*r^2*(a + b*x)*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)*(a + b*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^2*x^2 + 2*a*b*x + a^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

maxima [A] time = 0.89, size = 392, normalized size = 0.84

$$\frac{2\left(\frac{dfq\log(bx+a)}{bc-ad}-\frac{dfq\log(dx+c)}{bc-ad}-\frac{fp}{bx+a}\right)r\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bf}-\frac{\left(\frac{2\,df^2pq\log(dx+c)}{bc-ad}+\frac{2\left(pq+q^2\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+\right.\right.}{bc-ad}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] 2*(d*f*q*log(b*x + a)/(b*c - a*d) - d*f*q*log(d*x + c)/(b*c - a*d) - f*p/(b
*x + a))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - (2*d*f^2*p*q*log(d*
x + c)/(b*c - a*d) + 2*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d*f^2/(b*c - a*d) + (2*b*c*f^
2*p^2 - 2*a*d*f^2*p^2 + (b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)^2 - 2*(b
*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)*log(d*x + c) - (b*d*f^2*q^2*x + a*
d*f^2*q^2)*log(d*x + c)^2 - 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a))/(
a*b*c - a^2*d + (b^2*c - a*b*d)*x))*r^2/(b*f^2) - log(((b*x + a)^p*(d*x + c
)^q*f)^r*e)^2/((b*x + a)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**2,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**2, x)
```

3.22 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=632

$$-\frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}-\frac{d^2pqr^2\mathrm{Li}_2\left(\frac{b(c-}{bc-}\right.}{b(bc-ad)^2}$$

```
[Out] -1/4*p^2*r^2/b/(b*x+a)^2-3/2*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*p*q*r^2
*ln(b*x+a)/b/(-a*d+b*c)^2+d^2*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^2+1/2*d^2*p*q*
r^2*ln(b*x+a)^2/b/(-a*d+b*c)^2+1/2*d^2*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2
*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(
d*x+c)/b/(-a*d+b*c)^2-1/2*d^2*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^2+d^2*q^2*r^
2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^2-1/2*p*r*ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r)/b/(b*x+a)^2-d*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+
b*c)/(b*x+a)-d^2*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)
^2+d^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2-1/2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^2+d^2*q^2*r^2*polylog(2,-d*(b*x+a)
/(-a*d+b*c))/b/(-a*d+b*c)^2-d^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(
-a*d+b*c)^2
```

Rubi **[A]** time = 0.49, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$-\frac{d^2pqr^2\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}+\frac{d^2q^2r^2\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2q}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -(p^2*r^2)/(4*b*(a + b*x)^2) - (3*d*p*q*r^2)/(2*b*(b*c - a*d)*(a + b*x)) -
(d^2*p*q*r^2*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q^2*r^2*Log[a + b*x])
/(b*(b*c - a*d)^2) + (d^2*p*q*r^2*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2) + (d^
2*p*q*r^2*Log[c + d*x])/(2*b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x])/(b
*(b*c - a*d)^2) - (d^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*
x])/(b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2) +
(d^2*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
 - (p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(a + b*x)^2) - (d*q*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*Lo
g[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) + (d^2*q
```

```
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*b*(a + b*x)^2) + (d^2*q^2*r^2*Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
```

```
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + (pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}$$

$$= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + $$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(\right.\right.}{2b(a+bx)}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log}{2b(bc-}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log}{2b(bc-}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(}{b(bc-}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(}{b(bc-}$$

Mathematica [A] time = 1.33, size = 872, normalized size = 1.38

$$-\frac{c^2p^2r^2b^2 + 2d^2q^2r^2x^2\log^2(c+dx)b^2 + 2c^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)b^2 + 6cdpqr^2xb^2 + 4d^2q^2r^2x^2\log(c-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*q
*r^2 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*d^
```

```
2*p*q*r^2*(a + b*x)^2*Log[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*Log[c + d*x] + 4*a
^2*d^2*q^2*r^2*Log[c + d*x] - 4*a*b*d^2*p*q*r^2*x*Log[c + d*x] + 8*a*b*d^2*
q^2*r^2*x*Log[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*Log[c + d*x] + 4*b^2*d^2*q^2
*r^2*x^2*Log[c + d*x] + 2*a^2*d^2*q^2*r^2*Log[c + d*x]^2 + 4*a*b*d^2*q^2*r^
2*x*Log[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*Log[c + d*x]^2 - 2*d^2*q*r*(a +
b*x)^2*Log[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*Log[c + d*x] + 2*(p + q)*r*Log[
(b*(c + d*x))/(b*c - a*d)] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*b^
2*c^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r] + 4*a*b*c*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 4*a*b*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^
2*d^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r
*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^2*d^2*q*r*x^2*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2
+ 2*a^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d^2*q*(p + q)*r^2*(a
 + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^2*(a + b
*x)^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{(bx+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)^{2}}{(bx+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

maxima [A] time = 1.10, size = 755, normalized size = 1.19

$$-\frac{\left(\frac{2\,d^{2}fq\log(bx+a)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}-\frac{2\,d^{2}fq\log(dx+c)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}+\frac{2\,bdfqx-adf\left(p-2\,q\right)+bcfp}{a^{2}bc-a^{3}d+\left(b^{3}c-ab^{2}d\right)x^{2}+2\left(ab^{2}c-a^{2}bd\right)x}\right)r\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{2\,bf}+\frac{\left(\frac{4\left(pq+q^{2}\right)\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxima
")
[Out] -1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*lo
g(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*q)
 + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*b*d
)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2*f^2*l
og(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(p^2 - 3
*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*x^2 + 2*
a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f^2*p*q*x^
2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x + c) + 2*(b
^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(d*x + c)^2
+ 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^2*d^2*f^2*x^
2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)*log(b*x + a)
)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/(b*f^2) - 1/2*l
og(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)^{2}}{(a+b\,x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\left(a+bx\right)^{p}\left(c+dx\right)^{q}\right)^{r}\right)^{2}}{\left(a+bx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**3, x)
```

3.23 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx$

Optimal. Leaf size=764

$$\frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-\frac{2d^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}+\frac{2d^3pqr^2\text{Li}_2(}{3b(bc-}$$

```
[Out] -2/27*p^2*r^2/b/(b*x+a)^3-5/18*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^2+8/9*d^2*p*q
*r^2/b/(-a*d+b*c)^2/(b*x+a)-1/3*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)+2/9*d^3*
p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-d^3*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d
^3*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^3-2/9*d^3*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c
)^3+d^3*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*ln(-d*(b*x+a)/(-a*
d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^3+1/3*d^3*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^3
-2/3*d^3*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3-2/9*p*r*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3-1/3*d*q*r*ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^2+2/3*d^2*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b/(-a*d+b*c)^2/(b*x+a)+2/3*d^3*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)^3-2/3*d^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
/(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^3-2/3*d^3*q^2
*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*polylo
g(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3
```

Rubi [A] time = 0.61, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2d^3pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}-\frac{2d^3q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] (-2*p^2*r^2)/(27*b*(a + b*x)^3) - (5*d*p*q*r^2)/(18*b*(b*c - a*d)*(a + b*x)
^2) + (8*d^2*p*q*r^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (d^2*q^2*r^2)/(3*b*(b
*c - a*d)^2*(a + b*x)) + (2*d^3*p*q*r^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) -
 (d^3*q^2*r^2*Log[a + b*x])/(b*(b*c - a*d)^3) - (d^3*p*q*r^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3) - (2*d^3*p*q*r^2*Log[c + d*x])/(9*b*(b*c - a*d)^3) +
(d^3*q^2*r^2*Log[c + d*x])/(b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (d^3*q^2*r^2*Log[c
 + d*x]^2)/(3*b*(b*c - a*d)^3) - (2*d^3*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*
```

```
x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3) - (2*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b*(a + b*x)^3) - (d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3*q*r*Log[a + b*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - (2*d^3*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(3*b*(a + b*x)^3) - (2*d^3*q^2*r^2*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{(a+bx)^4}$$

$$= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \cdots$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p\right.\right.}{3b(a+bx}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{dpqr^2}{9b(bc-ad)(a+bx)^2} + \frac{2d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2}{9b(b}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr}{9b(}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{}{3b(bc-}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{}{3b(bc-}$$

Mathematica [A] time = 1.95, size = 1407, normalized size = 1.84

$$-\frac{4c^3p^2r^2b^3 + 18cd^2q^2r^2x^2b^3 - 48cd^2pqr^2x^2b^3 - 18d^3q^2r^2x^3\log^2(c+dx)b^3 + 18c^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] -1/54*(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2
- 4*a^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 63*
a^3*d^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c^2*
d*p*q*r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*a*b^
2*c*d^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2 + 48*
a*b^2*d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r^2*x^2
 + 18*d^3*p*q*r^2*(a + b*x)^3*Log[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*Log[c + d
*x] - 54*a^3*d^3*q^2*r^2*Log[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*Log[c + d*x]
 - 162*a^2*b*d^3*q^2*r^2*x*Log[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2*Log[c +
d*x] - 162*a*b^2*d^3*q^2*r^2*x^2*Log[c + d*x] + 12*b^3*d^3*p*q*r^2*x^3*Log[
c + d*x] - 54*b^3*d^3*q^2*r^2*x^3*Log[c + d*x] - 18*a^3*d^3*q^2*r^2*Log[c +
 d*x]^2 - 54*a^2*b*d^3*q^2*r^2*x*Log[c + d*x]^2 - 54*a*b^2*d^3*q^2*r^2*x^2*
Log[c + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*Log[c + d*x]^2 + 12*b^3*c^3*p*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 12*a^3*d^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*a*b^2*c^2*d*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r] + 54*a^3*d^3*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 18*b^3*c^2*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a*b^2*c*d^2*
q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*b*d^3*q*r*x*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r] - 36*b^3*c*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] + 36*a*b^2*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36
*a^3*d^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a^2*b*
d^3*q*r*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a*b^2*d^3
*q*r*x^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*b^3*d^3*q*r
*x^3*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^3*c^3*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 54*a*b^2*c^2*d*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 54*a^2*b*c*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 18*a
^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*d^3*q*r*(a + b*x)^3*Log[a
 + b*x]*(2*p*r - 9*q*r + 6*p*r*Log[c + d*x] - 6*(p + q)*r*Log[(b*(c + d*x))
/(b*c - a*d)] + 6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 36*d^3*q*(p + q)*
r^2*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^3*
(a + b*x)^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{b^{4}x^{4}+4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+4\,a^{3}bx+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="fricas
")

[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6*
a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac")

[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)

[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)

maxima [A] time = 1.73, size = 1252, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="maxima
")

[Out] 1/9*(6*d^3*f*q*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 6*d^3*f*q*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3) + (6*b^2*d^2*f*q*x^2 + a*b*c*d*f*(4*p - 3*q) - a^2*d^2*f*(2*p - 9*q)
 - 2*b^2*c^2*f*p - 3*(b^2*c*d*f*q - 5*a*b*d^2*f*q)*x)/(a^3*b^2*c^2 - 2*a^4*
b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2
- 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b
*d^2)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - 1/54*(36*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/

```
(b*c - a*d)))*d^3*f^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
 6*(2*p*q - 9*q^2)*d^3*f^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3) + (4*b^3*c^3*f^2*p^2 - 3*(4*p^2 - 5*p*q)*a*b^2*c^2*d*f^2 +
 6*(2*p^2 - 13*p*q + 3*q^2)*a^2*b*c*d^2*f^2 - (4*p^2 - 63*p*q + 18*q^2)*a^3
*d^3*f^2 - 6*((8*p*q - 3*q^2)*b^3*c*d^2*f^2 - (8*p*q - 3*q^2)*a*b^2*d^3*f^2
)*x^2 + 18*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2
*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)^2 - 36*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^
2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) - 18*(b^3*d^3*f^2*q^2*x^3 + 3*a*b^2*d^3*f^2*q^2*x^2 + 3*a^2*b*d
^3*f^2*q^2*x + a^3*d^3*f^2*q^2)*log(d*x + c)^2 + 3*(5*b^3*c^2*d*f^2*p*q - 6
*(7*p*q - 2*q^2)*a*b^2*c*d^2*f^2 + (37*p*q - 12*q^2)*a^2*b*d^3*f^2)*x - 6*(
(2*p*q - 9*q^2)*b^3*d^3*f^2*x^3 + 3*(2*p*q - 9*q^2)*a*b^2*d^3*f^2*x^2 + 3*(
2*p*q - 9*q^2)*a^2*b*d^3*f^2*x + (2*p*q - 9*q^2)*a^3*d^3*f^2)*log(b*x + a))
/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c
^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*
d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x))*r^2/(b*f^2) - 1/3*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)^2/((b*x + a)^3*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2}{(a+bx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**4, x)
```

3.24 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx$

Optimal. Leaf size=884

$$\frac{pqr^2\log^2(a+bx)d^4}{4b(bc-ad)^4}-\frac{q^2r^2\log^2(c+dx)d^4}{4b(bc-ad)^4}+\frac{11q^2r^2\log(a+bx)d^4}{12b(bc-ad)^4}-\frac{pqr^2\log(a+bx)d^4}{8b(bc-ad)^4}-\frac{11q^2r^2\log(c+dx)d^4}{12b(bc-ad)^4}+\frac{pqr^2}{8}$$

```
[Out] -1/32*p^2*r^2/b/(b*x+a)^4-7/72*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^3+3/16*d^2*p*
q*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-1/12*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-5/
8*d^3*p*q*r^2/b/(-a*d+b*c)^3/(b*x+a)+5/12*d^3*q^2*r^2/b/(-a*d+b*c)^3/(b*x+a
)-1/8*d^4*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^4+11/12*d^4*q^2*r^2*ln(b*x+a)/b/(-
a*d+b*c)^4+1/4*d^4*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^4+1/8*d^4*p*q*r^2*ln(d*
x+c)/b/(-a*d+b*c)^4-11/12*d^4*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^4-1/2*d^4*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*d^4*q^2*r^2*ln(d
*x+c)^2/b/(-a*d+b*c)^4+1/2*d^4*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b
/(-a*d+b*c)^4-1/8*p*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^4-1/6*d*q*r
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^3+1/4*d^2*q*r*ln(e*(f
*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2/(b*x+a)^2-1/2*d^3*q*r*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^3/(b*x+a)-1/2*d^4*q*r*ln(b*x+a)*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^4+1/2*d^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+
a)^4+1/2*d^4*q^2*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^4-1/2*d^
4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^4
```

Rubi [A] time = 0.74, antiderivative size = 884, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{pqr^2\log^2(a+bx)d^4}{4b(bc-ad)^4}-\frac{q^2r^2\log^2(c+dx)d^4}{4b(bc-ad)^4}+\frac{11q^2r^2\log(a+bx)d^4}{12b(bc-ad)^4}-\frac{pqr^2\log(a+bx)d^4}{8b(bc-ad)^4}-\frac{11q^2r^2\log(c+dx)d^4}{12b(bc-ad)^4}+\frac{pqr^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] -(p^2*r^2)/(32*b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)^3
) + (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*b*
(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b*x
])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^4)
 + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c +
```

```
d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*d)
^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(b*
c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2*r
^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^4*
q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^4)
+ (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a
*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(4*b*(a + b*x)^4) + (d^4*q^
2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^4*
p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
```

```
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{1}{2}(pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{(a+bx)^5}$$

$$= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \cdots$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e\left(f(a+bx)^p\right.\right.}{4b(a+b}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{dpqr^2}{24b(bc-ad)(a+bx)^3} + \frac{d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{}{8b(}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{}{8b(}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{}{12b}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{}{12b}$$

Mathematica [B] time = 3.00, size = 2003, normalized size = 2.27

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] (-9*b^4*c^4*p^2*r^2 + 36*a*b^3*c^3*d*p^2*r^2 - 54*a^2*b^2*c^2*d^2*p^2*r^2 +
 36*a^3*b*c*d^3*p^2*r^2 - 9*a^4*d^4*p^2*r^2 - 28*a*b^3*c^3*d*p*q*r^2 + 138*
a^2*b^2*c^2*d^2*p*q*r^2 - 372*a^3*b*c*d^3*p*q*r^2 + 262*a^4*d^4*p*q*r^2 - 2
4*a^2*b^2*c^2*d^2*q^2*r^2 + 168*a^3*b*c*d^3*q^2*r^2 - 144*a^4*d^4*q^2*r^2 -
 28*b^4*c^3*d*p*q*r^2*x + 192*a*b^3*c^2*d^2*p*q*r^2*x - 840*a^2*b^2*c*d^3*p
*q*r^2*x + 676*a^3*b*d^4*p*q*r^2*x - 48*a*b^3*c^2*d^2*q^2*r^2*x + 456*a^2*b
^2*c*d^3*q^2*r^2*x - 408*a^3*b*d^4*q^2*r^2*x + 54*b^4*c^2*d^2*p*q*r^2*x^2 -
 648*a*b^3*c*d^3*p*q*r^2*x^2 + 594*a^2*b^2*d^4*p*q*r^2*x^2 - 24*b^4*c^2*d^2
*q^2*r^2*x^2 + 408*a*b^3*c*d^3*q^2*r^2*x^2 - 384*a^2*b^2*d^4*q^2*r^2*x^2 -
180*b^4*c*d^3*p*q*r^2*x^3 + 180*a*b^3*d^4*p*q*r^2*x^3 + 120*b^4*c*d^3*q^2*r
^2*x^3 - 120*a*b^3*d^4*q^2*r^2*x^3 + 72*d^4*p*q*r^2*(a + b*x)^4*Log[a + b*x
]^2 + 36*a^4*d^4*p*q*r^2*Log[c + d*x] - 264*a^4*d^4*q^2*r^2*Log[c + d*x] +
144*a^3*b*d^4*p*q*r^2*x*Log[c + d*x] - 1056*a^3*b*d^4*q^2*r^2*x*Log[c + d*x
] + 216*a^2*b^2*d^4*p*q*r^2*x^2*Log[c + d*x] - 1584*a^2*b^2*d^4*q^2*r^2*x^2
*Log[c + d*x] + 144*a*b^3*d^4*p*q*r^2*x^3*Log[c + d*x] - 1056*a*b^3*d^4*q^2
*r^2*x^3*Log[c + d*x] + 36*b^4*d^4*p*q*r^2*x^4*Log[c + d*x] - 264*b^4*d^4*q
^2*r^2*x^4*Log[c + d*x] - 72*a^4*d^4*q^2*r^2*Log[c + d*x]^2 - 288*a^3*b*d^4
*q^2*r^2*x*Log[c + d*x]^2 - 432*a^2*b^2*d^4*q^2*r^2*x^2*Log[c + d*x]^2 - 28
8*a*b^3*d^4*q^2*r^2*x^3*Log[c + d*x]^2 - 72*b^4*d^4*q^2*r^2*x^4*Log[c + d*x
]^2 + 12*d^4*q*r*(a + b*x)^4*Log[a + b*x]*(-3*p*r + 22*q*r - 12*p*r*Log[c +
 d*x] + 12*(p + q)*r*Log[(b*(c + d*x))/(b*c - a*d)] - 12*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]) - 36*b^4*c^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 144*a*b^3*c^3*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 216*a^2*b^2*c^2
*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^3*b*c*d^3*p*r*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a^4*d^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r] - 48*a*b^3*c^3*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 216*a^
2*b^2*c^2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 432*a^3*b*c*d^3*q*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 264*a^4*d^4*q*r*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r] - 48*b^4*c^3*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] + 288*a*b^3*c^2*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 864*a^2*
b^2*c*d^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 624*a^3*b*d^4*q*r*x*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^4*c^2*d^2*q*r*x^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r] - 576*a*b^3*c*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + 504*a^2*b^2*d^4*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 144*b^4*c*d^3*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a*b^3*d^
4*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^4*d^4*q*r*Log[c + d*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a^3*b*d^4*q*r*x*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 864*a^2*b^2*d^4*q*r*x^2*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a*b^3*d^4*q*r*x^3*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*b^4*d^4*q*r*x^4*Log[c + d*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*b^4*c^4*Log[e*(f*(a + b*x)^p*(c + d*
```

```
x)^q)^r]^2 + 288*a*b^3*c^3*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 432*a
^2*b^2*c^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 288*a^3*b*c*d^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 72*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 144*d^4*q*(p + q)*r^2*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(288*b*(b*c - a*d)^4*(a + b*x)^4)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{b^5x^5 + 5\,ab^4x^4 + 10\,a^2b^3x^3 + 10\,a^3b^2x^2 + 5\,a^4bx + a^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^5*x^5 + 5*a*b^4*x^4 + 10
*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

maxima [B] time = 1.67, size = 1816, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] -1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3 -
a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p - 2
2*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^3*c^2
*d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*
c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^
2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a
^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(144*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*log(d*x + c)/(b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (9*b^4*c^4*f^2
*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*q + 4*q^2)*a^2*b
^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*f^2 + (9*p^2 - 26
2*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*d^3*f^2 - (3*p*q -
 2*q^2)*a*b^3*d^4*f^2)*x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*d^2*f^2 - 4*(27*p*q
 - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b^2*d^4*f^2)*x^2 - 72*(b
^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 +
4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)^2 + 144*(b^4*d^4*f^2*
p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4
*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) + 72*(b^4*d^4*f^2*q
^2*x^4 + 4*a*b^3*d^4*f^2*q^2*x^3 + 6*a^2*b^2*d^4*f^2*q^2*x^2 + 4*a^3*b*d^4*
f^2*q^2*x + a^4*d^4*f^2*q^2)*log(d*x + c)^2 + 4*(7*b^4*c^3*d*f^2*p*q - 12*(
4*p*q - q^2)*a*b^3*c^2*d^2*f^2 + 6*(35*p*q - 19*q^2)*a^2*b^2*c*d^3*f^2 - (1
69*p*q - 102*q^2)*a^3*b*d^4*f^2)*x + 12*((3*p*q - 22*q^2)*b^4*d^4*f^2*x^4 +
 4*(3*p*q - 22*q^2)*a*b^3*d^4*f^2*x^3 + 6*(3*p*q - 22*q^2)*a^2*b^2*d^4*f^2*
x^2 + 4*(3*p*q - 22*q^2)*a^3*b*d^4*f^2*x + (3*p*q - 22*q^2)*a^4*d^4*f^2)*lo
g(b*x + a))/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*
d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*
d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2
 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d +
6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4
*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x))*r^2/(
b*f^2) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^4*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)
[Out] Timed out
```

3.25 $\int (g+hx)^4 \log \left(e \left(f(a+bx)^p (c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=334

$$-\frac{pr(bg-ah)^5 \log(a+bx)}{5b^5h}-\frac{prx(bg-ah)^4}{5b^4}-\frac{pr(g+hx)^2(bg-ah)^3}{10b^3h}-\frac{pr(g+hx)^3(bg-ah)^2}{15b^2h}+\frac{(g+hx)^5 \log \left(e \left(f(a+\right.\right.}{5h}$$

```
[Out] -1/5*(-a*h+b*g)^4*p*r*x/b^4-1/5*(-c*h+d*g)^4*q*r*x/d^4-1/10*(-a*h+b*g)^3*p*
r*(h*x+g)^2/b^3/h-1/10*(-c*h+d*g)^3*q*r*(h*x+g)^2/d^3/h-1/15*(-a*h+b*g)^2*p
*r*(h*x+g)^3/b^2/h-1/15*(-c*h+d*g)^2*q*r*(h*x+g)^3/d^2/h-1/20*(-a*h+b*g)*p*
r*(h*x+g)^4/b/h-1/20*(-c*h+d*g)*q*r*(h*x+g)^4/d/h-1/25*p*r*(h*x+g)^5/h-1/25
*q*r*(h*x+g)^5/h-1/5*(-a*h+b*g)^5*p*r*ln(b*x+a)/b^5/h-1/5*(-c*h+d*g)^5*q*r*
ln(d*x+c)/d^5/h+1/5*(h*x+g)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] time = 0.19, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^4}{5b^4}-\frac{pr(g+hx)^2(bg-ah)^3}{10b^3h}-\frac{pr(g+hx)^3(bg-ah)^2}{15b^2h}-\frac{pr(bg-ah)^5 \log(a+bx)}{5b^5h}+\frac{(g+hx)^5 \log \left(e \left(f(a+\right.\right.}{5h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)^4*p*r*x)/(5*b^4) - ((d*g - c*h)^4*q*r*x)/(5*d^4) - ((b*g - a*
h)^3*p*r*(g + h*x)^2)/(10*b^3*h) - ((d*g - c*h)^3*q*r*(g + h*x)^2)/(10*d^3*
h) - ((b*g - a*h)^2*p*r*(g + h*x)^3)/(15*b^2*h) - ((d*g - c*h)^2*q*r*(g + h
*x)^3)/(15*d^2*h) - ((b*g - a*h)*p*r*(g + h*x)^4)/(20*b*h) - ((d*g - c*h)*q
*r*(g + h*x)^4)/(20*d*h) - (p*r*(g + h*x)^5)/(25*h) - (q*r*(g + h*x)^5)/(25
*h) - ((b*g - a*h)^5*p*r*Log[a + b*x])/(5*b^5*h) - ((d*g - c*h)^5*q*r*Log[c
 + d*x])/(5*d^5*h) + ((g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*
h)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(g+hx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h} - \frac{(bpr)\int \frac{(g+hx)^5}{a+bx}\,dx}{5h} - \frac{(}{}$$

$$= \frac{(g+hx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^4}{b^5} + \frac{(}{b}\right.}{}$$

$$= -\frac{(bg-ah)^4 prx}{5b^4} - \frac{(dg-ch)^4 qrx}{5d^4} - \frac{(bg-ah)^3 pr(g+hx)^2}{10b^3h} - \frac{(dg}{}$$

Mathematica [A] time = 0.33, size = 275, normalized size = 0.82

$$\frac{-\frac{pr\left(15b^4(g+hx)^4(bg-ah)+20b^3(g+hx)^3(bg-ah)^2+30b^2(g+hx)^2(bg-ah)^3+60bhx(bg-ah)^4+60(bg-ah)^5\log(a+bx)+12b^5(g+hx)^5\right)}{60b^5} + (g+hx)^5 \log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 20
*b^3*(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*(g
 + h*x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/b^5 - (q*r*(60*d*h*(d*g - c*h)^
4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)^3 +
 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^5*Log
[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5
*h)
```

fricas [B] time = 0.42, size = 945, normalized size = 2.83

$$-\frac{12\left(b^5d^5h^4p + b^5d^5h^4q\right)rx^5 + 15\left(\left(5\,b^5d^5gh^3 - ab^4d^5h^4\right)p + \left(5\,b^5d^5gh^3 - b^5cd^4h^4\right)q\right)rx^4 + 20\left(\left(10\,b^5d^5g^2h^2 - 5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r*x^5 + 15*((5*b^5*d^5*g*h^3 - a
*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b^5*c*d^4*h^4)*q)*r*x^4 + 20*((10*b^5*
d^5*g^2*h^2 - 5*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h^2
- 5*b^5*c*d^4*g*h^3 + b^5*c^2*d^3*h^4)*q)*r*x^3 + 30*((10*b^5*d^5*g^3*h - 1
0*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*b^5*d^
5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^4)*q)*
r*x^2 + 60*((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 10*a^2*b^3*d^5*g^2*h^2 -
5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5*g^4 - 10*b^5*c*d^4*g^3*
h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 + b^5*c^4*d*h^4)*q)*r*x -
60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4 + 10*b^5*d^5*g^2*h^2*p*r*
x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r*x + (5*a*b^4*d^5*g^4 - 1
0*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a^4*b*d^5*g*h^3 + a^5*d^5*
h^4)*p*r)*log(b*x + a) - 60*(b^5*d^5*h^4*q*r*x^5 + 5*b^5*d^5*g*h^3*q*r*x^4
+ 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*r*x^2 + 5*b^5*d^5*g^4*q*r
*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*b^5*c^3*d^2*g^2*h^2 - 5*b
^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) - 60*(b^5*d^5*h^4*x^5 + 5*b
^5*d^5*g*h^3*x^4 + 10*b^5*d^5*g^2*h^2*x^3 + 10*b^5*d^5*g^3*h*x^2 + 5*b^5*d^
5*g^4*x)*log(e) - 60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3*r*x^4 + 10*b^5*d^
5*g^2*h^2*r*x^3 + 10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4*r*x)*log(f))/(b^5*
d^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{4} \ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [B] time = 0.79, size = 624, normalized size = 1.87

$$\frac{1}{5}\left(h^4x^5+5\,gh^3x^4+10\,g^2h^2x^3+10\,g^3hx^2+5\,g^4x\right)\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)+\frac{r\left(\frac{60\left(5\,ab^4fg^4p-10\,a^2b^3fg^3hp+10\,a^3b^2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(h^4*x^5 + 5*g*h^3*x^4 + 10*g^2*h^2*x^3 + 10*g^3*h*x^2 + 5*g^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*r*(60*(5*a*b^4*f*g^4*p - 10*a^2*b^3
*f*g^3*h*p + 10*a^3*b^2*f*g^2*h^2*p - 5*a^4*b*f*g*h^3*p + a^5*f*h^4*p)*log(
b*x + a)/b^5 + 60*(5*c*d^4*f*g^4*q - 10*c^2*d^3*f*g^3*h*q + 10*c^3*d^2*f*g^
2*h^2*q - 5*c^4*d*f*g*h^3*q + c^5*f*h^4*q)*log(d*x + c)/d^5 - (12*b^4*d^4*f
*h^4*(p + q)*x^5 - 15*(a*b^3*d^4*f*h^4*p - (5*d^4*f*g*h^3*(p + q) - c*d^3*f
*h^4*q)*b^4)*x^4 - 20*(5*a*b^3*d^4*f*g*h^3*p - a^2*b^2*d^4*f*h^4*p - (10*d^
4*f*g^2*h^2*(p + q) - 5*c*d^3*f*g*h^3*q + c^2*d^2*f*h^4*q)*b^4)*x^3 - 30*(1
0*a*b^3*d^4*f*g^2*h^2*p - 5*a^2*b^2*d^4*f*g*h^3*p + a^3*b*d^4*f*h^4*p - (10
*d^4*f*g^3*h*(p + q) - 10*c*d^3*f*g^2*h^2*q + 5*c^2*d^2*f*g*h^3*q - c^3*d*f
*h^4*q)*b^4)*x^2 - 60*(10*a*b^3*d^4*f*g^3*h*p - 10*a^2*b^2*d^4*f*g^2*h^2*p
+ 5*a^3*b*d^4*f*g*h^3*p - a^4*d^4*f*h^4*p - (5*d^4*f*g^4*(p + q) - 10*c*d^3
*f*g^3*h*q + 10*c^2*d^2*f*g^2*h^2*q - 5*c^3*d*f*g*h^3*q + c^4*f*h^4*q)*b^4)
*x)/(b^4*d^4))/f
```

mupad [B] time = 1.03, size = 1128, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^4*x + (h^4*x^5)/5 + 2*g^3*h*x^2 + g
*h^3*x^4 + 2*g^2*h^2*x^3) - x^2*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r
*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a
*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q +
 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(10*b*d) - (a*c*((h^3*r*
(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*
d + 5*b*c))/(25*b*d)))/(2*b*d) + (g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*
d*g*q))/(b*d)) - x^4*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(
20*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(100*b*d)) - x*((a*c*(((5*a*d + 5
*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r
*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*
p + a*d*h*q + 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(b*d) - ((5
*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*
g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b
*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(b*d)
 + (a*c*h^4*r*(p + q))/(5*b*d)))/(5*b*d) - (a*c*((h^3*r*(b*c*h*p + 5*b*d*g*
p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d
)))/(b*d) + (2*g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*d*g*q))/(b*d)))/(5*
b*d) + (g^3*r*(2*b*c*h*p + b*d*g*p + 2*a*d*h*q + b*d*g*q))/(b*d)) + x^3*(((
```

```
5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d)
 - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(15*b*d) - (g*h^2*r*(b*c*h*p
+ 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(3*b*d) + (a*c*h^4*r*(p + q))/(15*b*d))
 + (log(a + b*x)*((a^5*h^4*p*r)/5 + a*b^4*g^4*p*r + 2*a^3*b^2*g^2*h^2*p*r -
 a^4*b*g*h^3*p*r - 2*a^2*b^3*g^3*h*p*r))/b^5 + (log(c + d*x)*((c^5*h^4*q*r)
/5 + c*d^4*g^4*q*r + 2*c^3*d^2*g^2*h^2*q*r - c^4*d*g*h^3*q*r - 2*c^2*d^3*g^
3*h*q*r))/d^5 - (h^4*r*x^5*(p + q))/25
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.26 $\int (g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=276

$$-\frac{pr(bg-ah)^4\log(a+bx)}{4b^4h}-\frac{prx(bg-ah)^3}{4b^3}-\frac{pr(g+hx)^2(bg-ah)^2}{8b^2h}+\frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}-pr($$

```
[Out] -1/4*(-a*h+b*g)^3*p*r*x/b^3-1/4*(-c*h+d*g)^3*q*r*x/d^3-1/8*(-a*h+b*g)^2*p*r
*(h*x+g)^2/b^2/h-1/8*(-c*h+d*g)^2*q*r*(h*x+g)^2/d^2/h-1/12*(-a*h+b*g)*p*r*(
h*x+g)^3/b/h-1/12*(-c*h+d*g)*q*r*(h*x+g)^3/d/h-1/16*p*r*(h*x+g)^4/h-1/16*q*
r*(h*x+g)^4/h-1/4*(-a*h+b*g)^4*p*r*ln(b*x+a)/b^4/h-1/4*(-c*h+d*g)^4*q*r*ln(
d*x+c)/d^4/h+1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] time = 0.13, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^3}{4b^3}-\frac{pr(g+hx)^2(bg-ah)^2}{8b^2h}-\frac{pr(bg-ah)^4\log(a+bx)}{4b^4h}+\frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}-pr($$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)^3*p*r*x)/(4*b^3) - ((d*g - c*h)^3*q*r*x)/(4*d^3) - ((b*g - a*
h)^2*p*r*(g + h*x)^2)/(8*b^2*h) - ((d*g - c*h)^2*q*r*(g + h*x)^2)/(8*d^2*h)
 - ((b*g - a*h)*p*r*(g + h*x)^3)/(12*b*h) - ((d*g - c*h)*q*r*(g + h*x)^3)/(
12*d*h) - (p*r*(g + h*x)^4)/(16*h) - (q*r*(g + h*x)^4)/(16*h) - ((b*g - a*h
)^4*p*r*Log[a + b*x])/(4*b^4*h) - ((d*g - c*h)^4*q*r*Log[c + d*x])/(4*d^4*h
) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
```

```
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4}{a+bx}dx}{4h} - \frac{(d$$

$$= \frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int\left(\frac{h(bg-ah)^3}{b^4} + \frac{(bg}{b^4(}\right.}{}$$

$$= -\frac{(bg-ah)^3prx}{4b^3} - \frac{(dg-ch)^3qrx}{4d^3} - \frac{(bg-ah)^2pr(g+hx)^2}{8b^2h} - \frac{(dg-}{}$$

Mathematica [A] time = 0.29, size = 231, normalized size = 0.84

$$\frac{\frac{1}{12}r\left(-\frac{p\left(4b^3(g+hx)^3(bg-ah)+6b^2(g+hx)^2(bg-ah)^2+12bhx(bg-ah)^3+12(bg-ah)^4\log(a+bx)+3b^4(g+hx)^4\right)}{b^4} - \frac{q\left(4d^3(g+hx)^3(dg-ch)+6d^2(g+hx)^2(dg-c\right.}{}\right.}{4h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(-((p*(12*b*h*(b*g - a*h)^3*x + 6*b^2*(b*g - a*h)^2*(g + h*x)^2 + 4*b^3
*(b*g - a*h)*(g + h*x)^3 + 3*b^4*(g + h*x)^4 + 12*(b*g - a*h)^4*Log[a + b*x
]))/b^4) - (q*(12*d*h*(d*g - c*h)^3*x + 6*d^2*(d*g - c*h)^2*(g + h*x)^2 + 4
*d^3*(d*g - c*h)*(g + h*x)^3 + 3*d^4*(g + h*x)^4 + 12*(d*g - c*h)^4*Log[c +
 d*x]))/d^4))/12 + (g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

fricas [B] time = 0.43, size = 679, normalized size = 2.46

$$-\frac{3\left(b^4d^4h^3p + b^4d^4h^3q\right)rx^4 + 4\left(\left(4\,b^4d^4gh^2 - ab^3d^4h^3\right)p + \left(4\,b^4d^4gh^2 - b^4cd^3h^3\right)q\right)rx^3 + 6\left(\left(6\,b^4d^4g^2h - 4\,ab^3d^4g\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r*x^4 + 4*((4*b^4*d^4*g*h^2 - a*b^
3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*c*d^3*h^3)*q)*r*x^3 + 6*((6*b^4*d^4*g
^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^2*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*b^4*c*
```

d^3*g*h^2 + b^4*c^2*d^2*h^3)*q)*r*x^2 + 12*((4*b^4*d^4*g^3 - 6*a*b^3*d^4*g^2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*c*d^3*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r*x - 12*(b^4*d^4*h^3*p*r*x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*g^3*p*r*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3)*p*r)*log(b*x + a) - 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*q*r*x^3 + 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3 - 6*b^4*c^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*log(d*x + c) - 12*(b^4*d^4*h^3*x^4 + 4*b^4*d^4*g*h^2*x^3 + 6*b^4*d^4*g^2*h*x^2 + 4*b^4*d^4*g^3*x)*log(e) - 12*(b^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^4*d^4*g^2*h*r*x^2 + 4*b^4*d^4*g^3*r*x)*log(f))/(b^4*d^4)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{3} \ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)

[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)

maxima [A] time = 0.79, size = 431, normalized size = 1.56

$$\frac{1}{4}\left(h^3x^4+4\,gh^2x^3+6\,g^2hx^2+4\,g^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)+\frac{r\left(\frac{12\left(4\,ab^3fg^3p-6\,a^2b^2fg^2hp+4\,a^3bfgh^2p-a^4fh^3p\right)\log}{b^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")

[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*d^

```
3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d^2
*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^3*
f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b^2
*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(p
+ q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*d^
3))/f
```

mupad [B] time = 0.83, size = 641, normalized size = 2.32

$$\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(g^3\,x+\frac{3\,g^2\,h\,x^2}{2}+g\,h^2\,x^3+\frac{h^3\,x^4}{4}\right)-x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{h^2\,r\left(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h}{4\,b\,d}\right.}{4\,b\,d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^3,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^3*x + (h^3*x^4)/4 + (3*g^2*h*x^2)/2
 + g*h^2*x^3) - x*(((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((h^2*r*(b*c*h*p + 4*
b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(
16*b*d)))/(4*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*h*q + 3*b*d*g*q))
/(2*b*d) + (a*c*h^3*r*(p + q))/(4*b*d)))/(4*b*d) + (g^2*r*(3*b*c*h*p + 2*b*
d*g*p + 3*a*d*h*q + 2*b*d*g*q))/(2*b*d) - (a*c*((h^2*r*(b*c*h*p + 4*b*d*g*p
 + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(16*b*d)
))/(b*d)) - x^3*((h^2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(12*b*
d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(48*b*d)) + x^2*(((4*a*d + 4*b*c)*((h^
2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(
4*a*d + 4*b*c))/(16*b*d)))/(8*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*
h*q + 3*b*d*g*q))/(4*b*d) + (a*c*h^3*r*(p + q))/(8*b*d)) - (log(a + b*x)*(a
^4*h^3*p*r - 4*a*b^3*g^3*p*r - 4*a^3*b*g*h^2*p*r + 6*a^2*b^2*g^2*h*p*r))/(4
*b^4) - (log(c + d*x)*(c^4*h^3*q*r - 4*c*d^3*g^3*q*r - 4*c^3*d*g*h^2*q*r +
6*c^2*d^2*g^2*h*q*r))/(4*d^4) - (h^3*r*x^4*(p + q))/16
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.27 $\int (g+hx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=218

$$-\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}-\frac{prx(bg-ah)^2}{3b^2}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qr(a}{}$$

```
[Out] -1/3*(-a*h+b*g)^2*p*r*x/b^2-1/3*(-c*h+d*g)^2*q*r*x/d^2-1/6*(-a*h+b*g)*p*r*(
h*x+g)^2/b/h-1/6*(-c*h+d*g)*q*r*(h*x+g)^2/d/h-1/9*p*r*(h*x+g)^3/h-1/9*q*r*(
h*x+g)^3/h-1/3*(-a*h+b*g)^3*p*r*ln(b*x+a)/b^3/h-1/3*(-c*h+d*g)^3*q*r*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^2}{3b^2}-\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qrx(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)^2*p*r*x)/(3*b^2) - ((d*g - c*h)^2*q*r*x)/(3*d^2) - ((b*g - a*
h)*p*r*(g + h*x)^2)/(6*b*h) - ((d*g - c*h)*q*r*(g + h*x)^2)/(6*d*h) - (p*r*
(g + h*x)^3)/(9*h) - (q*r*(g + h*x)^3)/(9*h) - ((b*g - a*h)^3*p*r*Log[a + b
*x])/(3*b^3*h) - ((d*g - c*h)^3*q*r*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int \frac{(g+hx)^3}{a+bx}\,dx}{3h} - \frac{(d$$

$$= \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int\left(\frac{h(bg-ah)^2}{b^3} + \frac{(bg}{b^3(}\right.}{}$$

$$= -\frac{(bg-ah)^2prx}{3b^2} - \frac{(dg-ch)^2qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh} - \frac{(dg-}{}$$

Mathematica [A] time = 0.24, size = 209, normalized size = 0.96

$$\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(b\left(6a^2d^3h^3px-3abd^3hp\left(g^2+6ghx+h^2x^2\right)+b^2d\left(6c^2h^3qx-3cdhq\left(g^2+6ghx+h^2x^2\right)+d^2(p+q)\left(5g^3\right.\right.\right.\right.}{6b^3d^3}}{3h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*b
*d^3*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^2
+ 6*g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^
3*x^3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x])))/(b^3*d^3) + (g + h*x)^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

fricas [B] time = 0.44, size = 441, normalized size = 2.02

$$-\frac{2\left(b^3d^3h^2p + b^3d^3h^2q\right)rx^3 + 3\left(\left(3\,b^3d^3gh - ab^2d^3h^2\right)p + \left(3\,b^3d^3gh - b^3cd^2h^2\right)q\right)rx^2 + 6\left(\left(3\,b^3d^3g^2 - 3\,ab^2d^3gh +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r*x^3 + 3*((3*b^3*d^3*g*h - a*b^2*
d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h^2)*q)*r*x^2 + 6*((3*b^3*d^3*g^2 -
 3*a*b^2*d^3*g*h + a^2*b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h + b^
3*c^2*d*h^2)*q)*r*x - 6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 + 3*b^
3*d^3*g^2*p*r*x + (3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*r)*lo
```

```
g(b*x + a) - 6*(b^3*d^3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d^3*g^2
*q*r*x + (3*b^3*c*d^2*g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*log(d*x + c
) - 6*(b^3*d^3*h^2*x^3 + 3*b^3*d^3*g*h*x^2 + 3*b^3*d^3*g^2*x)*log(e) - 6*(b
^3*d^3*h^2*r*x^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*log(f))/(b^3*d^
3)
```

giac [A] time = 100.51, size = 357, normalized size = 1.64

$$-\frac{1}{9}\left(h^2pr + h^2qr - 3h^2r\log(f) - 3h^2\right)x^3 + \frac{1}{3}\left(h^2prx^3 + 3ghprx^2 + 3g^2prx\right)\log(bx+a) + \frac{1}{3}\left(h^2qrx^3 + 3ghqrx^2 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/9*(h^2*p*r + h^2*q*r - 3*h^2*r*log(f) - 3*h^2)*x^3 + 1/3*(h^2*p*r*x^3 +
3*g*h*p*r*x^2 + 3*g^2*p*r*x)*log(b*x + a) + 1/3*(h^2*q*r*x^3 + 3*g*h*q*r*x^
2 + 3*g^2*q*r*x)*log(d*x + c) - 1/6*(3*b*d*g*h*p*r - a*d*h^2*p*r + 3*b*d*g*
h*q*r - b*c*h^2*q*r - 6*b*d*g*h*r*log(f) - 6*b*d*g*h)*x^2/(b*d) + 1/3*(3*a*
b^2*g^2*p*r - 3*a^2*b*g*h*p*r + a^3*h^2*p*r)*log(b*x + a)/b^3 + 1/3*(3*c*d^
2*g^2*q*r - 3*c^2*d*g*h*q*r + c^3*h^2*q*r)*log(-d*x - c)/d^3 - 1/3*(3*b^2*d
^2*g^2*p*r - 3*a*b*d^2*g*h*p*r + a^2*d^2*h^2*p*r + 3*b^2*d^2*g^2*q*r - 3*b^
2*c*d*g*h*q*r + b^2*c^2*h^2*q*r - 3*b^2*d^2*g^2*r*log(f) - 3*b^2*d^2*g^2)*x
/(b^2*d^2)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^2 \ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [A] time = 0.92, size = 269, normalized size = 1.23

$$\frac{1}{3}\left(h^2x^3 + 3ghx^2 + 3g^2x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right) + \frac{r\left(\frac{6\left(3ab^2fg^2p - 3a^2bfghp + a^3fh^2p\right)\log(bx+a)}{b^3} + \frac{6\left(3cd^2fg^2q - 3c^2df\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b^
```

```
3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
(2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*g
^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f
```

mupad [B] time = 0.65, size = 328, normalized size = 1.50

$$x\left(\frac{\left(\frac{h\,r\left(b\,c\,h\,p+3\,b\,d\,g\,p+a\,d\,h\,q+3\,b\,d\,g\,q\right)}{3\,b\,d}-\frac{h^2\,r\left(p+q\right)\left(3\,a\,d+3\,b\,c\right)}{9\,b\,d}\right)\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}-\frac{g\,r\,\left(b\,c\,h\,p+b\,d\,g\,p+a\,d\,h\,q+b\,d\,g\,q\right)}{b\,d}+\frac{a}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^2,x)
```

```
[Out] x*((((h*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(3*b*d) - (h^2*r*(p
+ q)*(3*a*d + 3*b*c))/(9*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (g*r*(b*c*h*p + b
*d*g*p + a*d*h*q + b*d*g*q))/(b*d) + (a*c*h^2*r*(p + q))/(3*b*d)) - x^2*((h
*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(6*b*d) - (h^2*r*(p + q)*(3
*a*d + 3*b*c))/(18*b*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^2*x + (h
^2*x^3)/3 + g*h*x^2) + (log(a + b*x)*(a^3*h^2*p*r + 3*a*b^2*g^2*p*r - 3*a^2
*b*g*h*p*r))/(3*b^3) + (log(c + d*x)*(c^3*h^2*q*r + 3*c*d^2*g^2*q*r - 3*c^2
*d*g*h*q*r))/(3*d^3) - (h^2*r*x^3*(p + q))/9
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

3.28 $\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=160

$$-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-$$

```
[Out] -1/2*(-a*h+b*g)*p*r*x/b-1/2*(-c*h+d*g)*q*r*x/d-1/4*p*r*(h*x+g)^2/h-1/4*q*r*
(h*x+g)^2/h-1/2*(-a*h+b*g)^2*p*r*ln(b*x+a)/b^2/h-1/2*(-c*h+d*g)^2*q*r*ln(d*
x+c)/d^2/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] time = 0.07, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2495, 43}

$$-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)*p*r*x)/(2*b) - ((d*g - c*h)*q*r*x)/(2*d) - (p*r*(g + h*x)^2)/
(4*h) - (q*r*(g + h*x)^2)/(4*h) - ((b*g - a*h)^2*p*r*Log[a + b*x])/(2*b^2*h
) - ((d*g - c*h)^2*q*r*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(2*h)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int \frac{(g+hx)^2}{a+bx}dx}{2h} - \frac{(dq}{}$$

$$= \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int\left(\frac{h(bg-ah)}{b^2}+\frac{(bg-a}{b^2(a+}\right.}{2h}$$

$$= -\frac{(bg-ah)prx}{2b} - \frac{(dg-ch)qrx}{2d} - \frac{pr(g+hx)^2}{4h} - \frac{qr(g+hx)^2}{4h} - \frac{(bg}{}$$

Mathematica [A] time = 0.21, size = 120, normalized size = 0.75

$$-\frac{b\left(dx\left(r(-2adhp-2bchq+bd(p+q)(4g+hx))-2bd(2g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2bcqr(ch-2dg}{4b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/4*(2*a*d^2*(-2*b*g + a*h)*p*r*Log[a + b*x] + b*(2*b*c*(-2*d*g + c*h)*q*r
*Log[c + d*x] + d*x*(r*(-2*a*d*h*p - 2*b*c*h*q + b*d*(p + q)*(4*g + h*x)) -
 2*b*d*(2*g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])))/(b^2*d^2)
```

fricas [A] time = 0.41, size = 242, normalized size = 1.51

$$-\frac{\left(b^2d^2hp+b^2d^2hq\right)rx^2+2\left(\left(2\,b^2d^2g-abd^2h\right)p+\left(2\,b^2d^2g-b^2cdh\right)q\right)rx-2\left(b^2d^2hprx^2+2\,b^2d^2gprx+\left(2\,abd^2g\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*h*p + b^2*d^2*h*q)*r*x^2 + 2*((2*b^2*d^2*g - a*b*d^2*h)*p +
(2*b^2*d^2*g - b^2*c*d*h)*q)*r*x - 2*(b^2*d^2*h*p*r*x^2 + 2*b^2*d^2*g*p*r*x
 + (2*a*b*d^2*g - a^2*d^2*h)*p*r)*log(b*x + a) - 2*(b^2*d^2*h*q*r*x^2 + 2*b
^2*d^2*g*q*r*x + (2*b^2*c*d*g - b^2*c^2*h)*q*r)*log(d*x + c) - 2*(b^2*d^2*h
*x^2 + 2*b^2*d^2*g*x)*log(e) - 2*(b^2*d^2*h*r*x^2 + 2*b^2*d^2*g*r*x)*log(f)
)/(b^2*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

maxima [A] time = 0.57, size = 143, normalized size = 0.89

$$\frac{1}{2}\left(hx^{2}+2\,gx\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)+\frac{r\left(\frac{2\left(2\,abfgp-a^{2}fhp\right)\log(bx+a)}{b^{2}}+\frac{2\left(2\,cdfgq-c^{2}fhq\right)\log(dx+c)}{d^{2}}-\frac{bdfh(p+q)x^{2}-2\,(\cdots}{\cdots}\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*r*(2*(2*a*b*
f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x +
 c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*q)
*b)*x)/(b*d))/f
```

mupad [B] time = 0.51, size = 153, normalized size = 0.96

$$\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)\left(\frac{h\,x^{2}}{2}+g\,x\right)-x\left(\frac{r\,\left(b\,c\,h\,p+2\,b\,d\,g\,p+a\,d\,h\,q+2\,b\,d\,g\,q\right)}{2\,b\,d}-\frac{h\,r\,\left(p+q\right)\,(2\,a\,d+2\cdots}{4\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g*x + (h*x^2)/2) - x*((r*(b*c*h*p + 2
*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(2*b*d) - (h*r*(p + q)*(2*a*d + 2*b*c))/(4
*b*d)) - (log(a + b*x)*(a^2*h*p*r - 2*a*b*g*p*r))/(2*b^2) - (log(c + d*x)*(
c^2*h*q*r - 2*c*d*g*q*r))/(2*d^2) - (h*r*x^2*(p + q))/4
```

sympy [A] time = 75.96, size = 632, normalized size = 3.95

$$\begin{cases} \left(gx + \frac{hx^2}{2}\right)\log\left(e\left(a^pc^qf\right)^r\right) \\ -\frac{c^2hqr\log(c+dx)}{2d^2} + \frac{cgqr\log(c+dx)}{d} + \frac{chqrx}{2d} + gprx\log(a) + gqrx\log(c+dx) - gqrx + grx\log(f) + gx\log(e) + \frac{hprx^2 \cdots}{2 \cdots} \\ -\frac{a^2hpr\log(a+bx)}{2b^2} + \frac{agpr\log(a+bx)}{b} + \frac{ahprx}{2b} + gprx\log(a+bx) - gprx + gqrx\log(c) + grx\log(f) + gx\log(e) + \frac{hprx^2 \cdots}{\cdots} \\ -\frac{a^2hpr\log(a+bx)}{2b^2} - \frac{a^2hqr\log(c+dx)}{2b^2} + \frac{a^2hqr\log\left(\frac{c}{d}+x\right)}{2b^2} + \frac{agpr\log(a+bx)}{b} + \frac{agqr\log(c+dx)}{b} - \frac{agqr\log\left(\frac{c}{d}+x\right)}{b} + \frac{ahprx}{2b} - \frac{c^2hqr\log\left(\frac{c}{d}+x\right) \cdots}{2d^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise(((g*x + h*x**2/2)*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)),
(-c**2*h*q*r*log(c + d*x)/(2*d**2) + c*g*q*r*log(c + d*x)/d + c*h*q*r*x/(2*
d) + g*p*r*x*log(a) + g*q*r*x*log(c + d*x) - g*q*r*x + g*r*x*log(f) + g*x*l
og(e) + h*p*r*x**2*log(a)/2 + h*q*r*x**2*log(c + d*x)/2 - h*q*r*x**2/4 + h*
r*x**2*log(f)/2 + h*x**2*log(e)/2, Eq(b, 0)), (-a**2*h*p*r*log(a + b*x)/(2*
b**2) + a*g*p*r*log(a + b*x)/b + a*h*p*r*x/(2*b) + g*p*r*x*log(a + b*x) - g
*p*r*x + g*q*r*x*log(c) + g*r*x*log(f) + g*x*log(e) + h*p*r*x**2*log(a + b*
x)/2 - h*p*r*x**2/4 + h*q*r*x**2*log(c)/2 + h*r*x**2*log(f)/2 + h*x**2*log(
e)/2, Eq(d, 0)), (-a**2*h*p*r*log(a + b*x)/(2*b**2) - a**2*h*q*r*log(c + d*
x)/(2*b**2) + a**2*h*q*r*log(c/d + x)/(2*b**2) + a*g*p*r*log(a + b*x)/b + a
*g*q*r*log(c + d*x)/b - a*g*q*r*log(c/d + x)/b + a*h*p*r*x/(2*b) - c**2*h*q
*r*log(c/d + x)/(2*d**2) + c*g*q*r*log(c/d + x)/d + c*h*q*r*x/(2*d) + g*p*r
*x*log(a + b*x) - g*p*r*x + g*q*r*x*log(c + d*x) - g*q*r*x + g*r*x*log(f) +
 g*x*log(e) + h*p*r*x**2*log(a + b*x)/2 - h*p*r*x**2/4 + h*q*r*x**2*log(c +
 d*x)/2 - h*q*r*x**2/4 + h*r*x**2*log(f)/2 + h*x**2*log(e)/2, True))
```

3.29 $\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=61

$$\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))$$

```
[Out] -(p+q)*r*x+(-a*d+b*c)*q*r*ln(d*x+c)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b
```

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2487, 31, 8}

$$\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}+rx(-(p+q))$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rubi steps

$$\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} + \frac{((bc-ad)qr)\int \frac{1}{c+dx}dx}{b} - ((p+q$$

$$= -(p+q)rx + \frac{(bc-ad)qr\log(c+dx)}{bd} + \frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)\right.\right.}{b}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.93

$$x\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - r(p+q)\right) + \frac{apr\log(a+bx)}{b} + \frac{cqr\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a*p*r*Log[a + b*x])/b + (c*q*r*Log[c + d*x])/d + x*(-((p + q)*r) + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])
```

fricas [A] time = 0.41, size = 72, normalized size = 1.18

$$\frac{bdrx\log(f) + bdx\log(e) - \left(bdp + bdq\right)rx + \left(bdprx + adpr\right)\log(bx+a) + \left(bdqrx + bcqr\right)\log(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] (b*d*r*x*log(f) + b*d*x*log(e) - (b*d*p + b*d*q)*r*x + (b*d*p*r*x + a*d*p*r
)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(b*d)
```

giac [A] time = 0.20, size = 66, normalized size = 1.08

$$prx\log(bx+a) + qrx\log(dx+c) + \frac{apr\log(bx+a)}{b} + \frac{cqr\log(-dx-c)}{d} - \left(pr + qr - r\log(f) - 1\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] p*r*x*log(b*x + a) + q*r*x*log(d*x + c) + a*p*r*log(b*x + a)/b + c*q*r*log(
-d*x - c)/d - (p*r + q*r - r*log(f) - 1)*x
```

maple [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{apr\ln(bx+a)}{b} + \frac{cqr\ln(dx+c)}{d} - prx - qrx + x\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-r*p*x-r*q*x+r*c*q/d*ln(d*x+c)+r*a*p/b*ln(
b*x+a)
```

maxima [A] time = 0.62, size = 75, normalized size = 1.23

$$x \log\left(\left((bx+a)^p (dx+c)^q f\right)^r e\right) - \frac{\left(bfp\left(\frac{x}{b} - \frac{a\log(bx+a)}{b^2}\right) + dfq\left(\frac{x}{d} - \frac{c\log(dx+c)}{d^2}\right)\right)r}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) - (b*f*p*(x/b - a*log(b*x + a)/b^2)
+ d*f*q*(x/d - c*log(d*x + c)/d^2))*r/f
```

mupad [B] time = 0.22, size = 60, normalized size = 0.98

$$x\,\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right) - p\,r\,x - q\,r\,x + \frac{a\,p\,r\,\ln(a+b\,x)}{b} + \frac{c\,q\,r\,\ln(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r),x)
```

```
[Out] x*log(e*(f*(a + b*x)^p*(c + d*x)^q)^r) - p*r*x - q*r*x + (a*p*r*log(a + b*x
))/b + (c*q*r*log(c + d*x))/d
```

sympy [A] time = 17.64, size = 187, normalized size = 3.07

$$\begin{cases} x\log\left(e\left(a^p c^q f\right)^r\right) & \text{for } b = 0 \\ \frac{cqr\log(c+dx)}{d} + prx\log(a) + qrx\log(c+dx) - qrx + rx\log(f) + x\log(e) & \text{for } b = 0 \\ \frac{apr\log(a+bx)}{b} + prx\log(a+bx) - prx + qrx\log(c) + rx\log(f) + x\log(e) & \text{for } d = 0 \\ \frac{apr\log(a+bx)}{b} + \frac{cqr\log(c+dx)}{d} + prx\log(a+bx) - prx + qrx\log(c+dx) - qrx + rx\log(f) + x\log(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (c*q*r*log(c +
d*x)/d + p*r*x*log(a) + q*r*x*log(c + d*x) - q*r*x + r*x*log(f) + x*log(e),
 Eq(b, 0)), (a*p*r*log(a + b*x)/b + p*r*x*log(a + b*x) - p*r*x + q*r*x*log(
c) + r*x*log(f) + x*log(e), Eq(d, 0)), (a*p*r*log(a + b*x)/b + c*q*r*log(c
+ d*x)/d + p*r*x*log(a + b*x) - p*r*x + q*r*x*log(c + d*x) - q*r*x + r*x*lo
g(f) + x*log(e), True))
```

3.30 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

Optimal. Leaf size=148

$$\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}-\frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{pr\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h}-\frac{qr\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{h}-\frac{qr\log\ldots}{}$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h-q*r*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-p*r*polylog(2,b*(h*x+
g)/(-a*h+b*g))/h-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Rubi [A] time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2494, 2394, 2393, 2391}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}-\frac{pr\log(g+hx)\log\ldots}{h}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) - (q*r*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]*Log[g + h*x])/h - (p*r*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h - (q*r
*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(bpr)\int\frac{\log(g+hx)}{a+bx}\,dx}{h} - \frac{(dqr)\int}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f\right.\right.}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f\right.\right.}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e\left(f\right.\right.}{}$$

Mathematica [A] time = 0.10, size = 163, normalized size = 1.10

$$\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + pr\text{Li}_2\left(\frac{h(a+bx)}{ah-bg}\right) - pr\log(a+bx)\log(g+hx) + pr\log(a+bx)\log\left(\frac{b(g}{bg}\right.}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/h
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
```

maxima [A] time = 0.93, size = 186, normalized size = 1.26

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f}-\frac{\left(fp\log(bx+a)+fq\log(dx+c)\right.}{fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="maxima")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilo
g(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(d*x
 + c))*r*log(h*x + g)/(f*h) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(h*x
+ g)/h
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(g + h*x), x)
```

3.31 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2} dx$

Optimal. Leaf size=128

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)}-\frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

```
[Out] b*p*r*ln(b*x+a)/h/(-a*h+b*g)+d*q*r*ln(d*x+c)/h/(-c*h+d*g)-ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/h/(h*x+g)-b*p*r*ln(h*x+g)/h/(-a*h+b*g)-d*q*r*ln(h*x+g)/h/(-c
*h+d*g)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 36, 31}

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)}-\frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (b*p*r*Log[a + b*x])/(h*(b*g - a*h)) + (d*q*r*Log[c + d*x])/(h*(d*g - c*h))
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)) - (b*p*r*Log[g + h*x]
)/(h*(b*g - a*h)) - (d*q*r*Log[g + h*x])/(h*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
```

```
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)}\,dx}{h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+\cdots}}{h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{(bpr)\int \frac{1}{g+hx}\,dx}{bg-ah} + \frac{\left(b^2pr\right)\int \frac{1}{a+bx}\,dx}{h(bg-ah)} - \cdots$$

$$= \frac{bpr\log(a+bx)}{h(bg-ah)} + \frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{bpr\cdots}{h\cdots}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.73

$$\frac{-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} + \frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah} + \frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)) + (b*p*r*(Log[a + b*x] -
 Log[g + h*x]))/(b*g - a*h) + (d*q*r*(Log[c + d*x] - Log[g + h*x]))/(d*g -
c*h))/h
```

fricas [B] time = 109.51, size = 280, normalized size = 2.19

$$-\frac{\left(bdg^2 + ach^2 - (bc+ad)gh\right)r\log(f) - \left(\left(bdgh - bch^2\right)prx + \left(adgh - ach^2\right)pr\right)\log(bx+a) - \left(\left(bdgh - adh^2\right)qrx \cdots\right.}{bdg^3h \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] -((b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*r*log(f) - ((b*d*g*h - b*c*h^2)*p*r
*x + (a*d*g*h - a*c*h^2)*p*r)*log(b*x + a) - ((b*d*g*h - a*d*h^2)*q*r*x + (
b*c*g*h - a*c*h^2)*q*r)*log(d*x + c) + (((b*d*g*h - b*c*h^2)*p + (b*d*g*h -
```

```
 a*d*h^2)*q)*r*x + ((b*d*g^2 - b*c*g*h)*p + (b*d*g^2 - a*d*g*h)*q)*r)*log(h
*x + g) + (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(e))/(b*d*g^3*h + a*c*g*
h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

giac [A] time = 0.37, size = 190, normalized size = 1.48

$$\frac{b^2 pr\log\left(|-bx-a|\right)}{b^2gh-abh^2}+\frac{d^2qr\log\left(|dx+c|\right)}{d^2gh-cdh^2}-\frac{pr\log\left(bx+a\right)}{h^2x+gh}-\frac{qr\log\left(dx+c\right)}{h^2x+gh}-\frac{\left(bdgpr-bchpr+bdgqr-adhqr\right)\log}{bdg^2h-bcgh^2-adgh^2+ach}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] b^2*p*r*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) + d^2*q*r*log(abs(d*x + c))/
(d^2*g*h - c*d*h^2) - p*r*log(b*x + a)/(h^2*x + g*h) - q*r*log(d*x + c)/(h^
2*x + g*h) - (b*d*g*p*r - b*c*h*p*r + b*d*g*q*r - a*d*h*q*r)*log(h*x + g)/(
b*d*g^2*h - b*c*g*h^2 - a*d*g*h^2 + a*c*h^3) - (r*log(f) + 1)/(h^2*x + g*h)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
```

maxima [A] time = 0.72, size = 123, normalized size = 0.96

$$\frac{\left(bfp\left(\frac{\log(bx+a)}{bg-ah}-\frac{\log(hx+g)}{bg-ah}\right)+dfq\left(\frac{\log(dx+c)}{dg-ch}-\frac{\log(hx+g)}{dg-ch}\right)\right)r}{fh}-\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{\left(hx+g\right)h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] (b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(d
*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)^p
*(d*x + c)^q*f)^r*e)/((h*x + g)*h)
```

mupad [B] time = 1.20, size = 152, normalized size = 1.19

$$\frac{\ln\left(g+h\,x\right)\,\left(b\,c\,h\,p\,r-g\,\left(b\,d\,p\,r+b\,d\,q\,r\right)+a\,d\,h\,q\,r\right)}{a\,c\,h^3-a\,d\,g\,h^2-b\,c\,g\,h^2+b\,d\,g^2\,h}-\frac{\ln\left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\left(x+\frac{g}{h}\right)}{\left(g+h\,x\right)^2}-\frac{b\,p\,r\,\ln\left(a+b\,x\right)}{a\,h^2-b\,g\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^2,x)
```

```
[Out] (log(g + h*x)*(b*c*h*p*r - g*(b*d*p*r + b*d*q*r) + a*d*h*q*r))/(a*c*h^3 - a
*d*g*h^2 - b*c*g*h^2 + b*d*g^2*h) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(
x + g/h))/(g + h*x)^2 - (b*p*r*log(a + b*x))/(a*h^2 - b*g*h) - (d*q*r*log(c
 + d*x))/(c*h^2 - d*g*h)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)
```

```
[Out] Timed out
```

3.32 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=202

$$\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2}+\frac{bpr}{2h(g+hx)(bg-ah)}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{d^2}{2}$$

```
[Out] 1/2*b*p*r/h/(-a*h+b*g)/(h*x+g)+1/2*d*q*r/h/(-c*h+d*g)/(h*x+g)+1/2*b^2*p*r*l
n(b*x+a)/h/(-a*h+b*g)^2+1/2*d^2*q*r*ln(d*x+c)/h/(-c*h+d*g)^2-1/2*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^2-1/2*b^2*p*r*ln(h*x+g)/h/(-a*h+b*g)^2-1/2*
d^2*q*r*ln(h*x+g)/h/(-c*h+d*g)^2
```

Rubi [A] time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2}+\frac{bpr}{2h(g+hx)(bg-ah)}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{d^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (b*p*r)/(2*h*(b*g - a*h)*(g + h*x)) + (d*q*r)/(2*h*(d*g - c*h)*(g + h*x)) +
 (b^2*p*r*Log[a + b*x])/(2*h*(b*g - a*h)^2) + (d^2*q*r*Log[c + d*x])/(2*h*(
d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*h*(g + h*x)^2) - (b
^2*p*r*Log[g + h*x])/(2*h*(b*g - a*h)^2) - (d^2*q*r*Log[g + h*x])/(2*h*(d*g
 - c*h)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
```

```
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^2}\,dx}{2h} + \frac{(dqr)\int \frac{1}{(c+dx)(g}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(g+hx)^2} - \frac{}{(}\right.}{2h}$$

$$= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} + \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} + \frac{d^2qr\log}{2h(dg}$$

Mathematica [A] time = 0.53, size = 206, normalized size = 1.02

$$\frac{\frac{r(g+hx)\left((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(adq+bcp))-(g+hx)\left(d^2q(ad-bc)(bg-ah)^2(\log(c+dx)-\log(g+hx))-b^2p(bc-ad)(dg-ch)^2(\log(a+bx)-\log(g+h\right.\right.}{(bc-ad)(bg-ah)^2(dg-ch)^2}}{2h(g+hx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (-Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*c - a*d)*(b*g - a
*h)*(d*g - c*h)*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*(-(b^2*(b*c
 - a*d)*(d*g - c*h)^2*p*(Log[a + b*x] - Log[g + h*x])) + d^2*(-(b*c) + a*d)
*(b*g - a*h)^2*q*(Log[c + d*x] - Log[g + h*x]))))/((b*c - a*d)*(b*g - a*h)^
2*(d*g - c*h)^2))/(2*h*(g + h*x)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.57, size = 595, normalized size = 2.95

$$\frac{b^3pr\log(|bx+a|)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)} + \frac{d^3qr\log(|dx+c|)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)} - \frac{pr\log(bx+a)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} - \frac{qr\log(dx+c)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*p*r*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) + 1/2
*d^3*q*r*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*p*
r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2*c^2
*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(h*x +
 g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*g^2*h^
3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*d*g*h^4
 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p*r*x + b*d*g*h*q*r*x - a*d*
h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*q*r - a*d*g*h*q*r - b*d*g^2
*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f) - a*c*h^2*r*log(f) - b*d*g^
2 + b*c*g*h + a*d*g*h - a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h
^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x
+ 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

maxima [A] time = 0.78, size = 232, normalized size = 1.15

$$\frac{\left(bfp\left(\frac{b\log(bx+a)}{b^2g^2-2\,abgh+a^2h^2}-\frac{b\log\left(hx+g\right)}{b^2g^2-2\,abgh+a^2h^2}+\frac{1}{bg^2-agh+\left(bgh-ah^2\right)x}\right)+dfq\left(\frac{d\log(dx+c)}{d^2g^2-2\,cdgh+c^2h^2}-\frac{d\log\left(hx+g\right)}{d^2g^2-2\,cdgh+c^2h^2}+\frac{1}{dg^2-cgh+\left(dgh\right.}\right.\right.}{2\,fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*f*p*(b*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) - b*log(h*x + g)
/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + 1/(b*g^2 - a*g*h + (b*g*h - a*h^2)*x)) +
 d*f*q*(d*log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) - d*log(h*x + g)/(d^
2*g^2 - 2*c*d*g*h + c^2*h^2) + 1/(d*g^2 - c*g*h + (d*g*h - c*h^2)*x)))*r/(f
*h) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)^2*h)
```

mupad [B] time = 2.71, size = 384, normalized size = 1.90

$$\frac{b^2\,p\,r\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}-\frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(q\,r\,a^2\,d^2+p\,r\,b^2\,c^2\right)-h\,\left(2\,c\,g\,p\,r\,b^2\,d+2\,a\,g\,q\,r\,b\,d^2\right.\right.}{2\,a^2\,c^2\,h^5-4\,a^2\,c\,d\,g\,h^4+2\,a^2\,d^2\,g^2\,h^3-4\,a\,b\,c^2\,g\,h^4+8\,a\,b\,c\,d\,g^2\,h^3-4\,a\,b\,d^2\,g^3\,h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^3,x)
```

```
[Out] (b^2*p*r*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*a*b*g*h^2) - (log(g + h
*x)*(h^2*(b^2*c^2*p*r + a^2*d^2*q*r) - h*(2*a*b*d^2*g*q*r + 2*b^2*c*d*g*p*r
) + b^2*d^2*g^2*p*r + b^2*d^2*g^2*q*r))/(2*a^2*c^2*h^5 + 2*b^2*d^2*g^4*h +
2*a^2*d^2*g^2*h^3 + 2*b^2*c^2*g^2*h^3 - 4*a*b*c^2*g*h^4 - 4*a^2*c*d*g*h^4 -
 4*a*b*d^2*g^3*h^2 - 4*b^2*c*d*g^3*h^2 + 8*a*b*c*d*g^2*h^3) - (log(e*(f*(a
+ b*x)^p*(c + d*x)^q)^r)*(x/2 + g/(2*h)))/(g + h*x)^3 - (b*c*h*p*r - b*d*g*
p*r + a*d*h*q*r - b*d*g*q*r)/((2*g*h + 2*h^2*x)*(a*c*h^2 + b*d*g^2 - a*d*g*
h - b*c*g*h)) + (d^2*q*r*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h
^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)
```

```
[Out] Timed out
```

3.33 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=260

$$\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}+\frac{b^2pr}{3h(g+hx)(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{bpr}{6h(g+hx)^2(bg-ah}$$

```
[Out] 1/6*b*p*r/h/(-a*h+b*g)/(h*x+g)^2+1/6*d*q*r/h/(-c*h+d*g)/(h*x+g)^2+1/3*b^2*p
*r/h/(-a*h+b*g)^2/(h*x+g)+1/3*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*p*r*ln
(b*x+a)/h/(-a*h+b*g)^3+1/3*d^3*q*r*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^3-1/3*b^3*p*r*ln(h*x+g)/h/(-a*h+b*g)^3-1/3*d
^3*q*r*ln(h*x+g)/h/(-c*h+d*g)^3
```

Rubi [A] time = 0.15, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^2pr}{3h(g+hx)(bg-ah)^2}+\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{bpr}{6h(g+hx)^2(bg-ah}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (b*p*r)/(6*h*(b*g - a*h)*(g + h*x)^2) + (d*q*r)/(6*h*(d*g - c*h)*(g + h*x)^
2) + (b^2*p*r)/(3*h*(b*g - a*h)^2*(g + h*x)) + (d^2*q*r)/(3*h*(d*g - c*h)^2
*(g + h*x)) + (b^3*p*r*Log[a + b*x])/(3*h*(b*g - a*h)^3) + (d^3*q*r*Log[c +
 d*x])/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*h*(g +
 h*x)^3) - (b^3*p*r*Log[g + h*x])/(3*h*(b*g - a*h)^3) - (d^3*q*r*Log[g + h*
x])/(3*h*(d*g - c*h)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
```

```
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{(dqr)\int \frac{1}{(c+dx)(g}}{3h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int\left(\frac{b^3}{(bg-ah)^3(a+bx)} - \frac{h}{(bg-ah)(g+hx)^3} - \right.}{3h}$$

$$= \frac{bpr}{6h(bg-ah)(g+hx)^2} + \frac{dqr}{6h(dg-ch)(g+hx)^2} + \frac{b^2pr}{3h(bg-ah)^2(g+hx)} + \frac{}{3}$$

Mathematica [A] time = 0.76, size = 254, normalized size = 0.98

$$\frac{\frac{r(g+hx)\left((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(adq+bcp))-(g+hx)\left((bg-ah)(dg-ch)\left(-2a^2d^2h^2q+4abd^2ghq-2b^2\left(c^2h^2p-2cdghp+d^2g^2(p+q)\right)\right)-2(g+hx)\left(b^3p(dg-\right.\right.\right.}{(bg-ah)^3(dg-ch)^3}}{6h(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (-2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*g
 - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d*g
 - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h^2*
p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(Log[a + b*x] - Lo
g[g + h*x]) + d^3*(b*g - a*h)^3*q*(Log[c + d*x] - Log[g + h*x])))))/((b*g -
 a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.85, size = 1765, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3*b^3*
c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2*h*q*r
 + 3*a^2*b*d^3*g*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^6*h - 3
*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*a*b^2*c*
d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*d*g^3*h^4
 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5 + 9*a^2*b*
c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*c^2*d*g*h^6
 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*h^3*p*r*x^2
+ 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^2*g*h^3*q*r*x
^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c*d*g^2*h^2*p*r
*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c*d*g*h^3*p*r*x
- a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h^2*q*r*x - 10*a*
b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*h^3*q*r*x - a^2*c
*d*h^4*q*r*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r - a*b*d^2*g^3*h*p*r
+ 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^2*g*h^3*p*r + 3*b^2
*d^2*g^4*q*r - b^2*c*d*g^3*h*q*r - 6*a*b*d^2*g^3*h*q*r + 2*a*b*c*d*g^2*h^2*
q*r + 3*a^2*d^2*g^2*h^2*q*r - a^2*c*d*g*h^3*q*r - 2*b^2*d^2*g^4*r*log(f) +
4*b^2*c*d*g^3*h*r*log(f) + 4*a*b*d^2*g^3*h*r*log(f) - 2*b^2*c^2*g^2*h^2*r*l
og(f) - 8*a*b*c*d*g^2*h^2*r*log(f) - 2*a^2*d^2*g^2*h^2*r*log(f) + 4*a*b*c^2
*g*h^3*r*log(f) + 4*a^2*c*d*g*h^3*r*log(f) - 2*a^2*c^2*h^4*r*log(f) - 2*b^2
*d^2*g^4 + 4*b^2*c*d*g^3*h + 4*a*b*d^2*g^3*h - 2*b^2*c^2*g^2*h^2 - 8*a*b*c*
d*g^2*h^2 - 2*a^2*d^2*g^2*h^2 + 4*a*b*c^2*g*h^3 + 4*a^2*c*d*g*h^3 - 2*a^2*c
^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d^2*g^3*h^5*x^
3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*h^6*x^3 - 2*a
*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b^2*d^2*g^5*h^
3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*c^2*g^3*h^5*x
^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^2*g^2*h^6*x^2
 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6*h^2*x - 6*b^
2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x + 12*a*b*c*d*g^
4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*d*g^3*h^5*x +
 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a*b*d^2*g^6*h^
2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*a*b*c^2*g^4*h
^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{\left(hx+g\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

maxima [A] time = 0.76, size = 456, normalized size = 1.75

$$\frac{\left(\left(\frac{2\,b^{2}\log(bx+a)}{b^{3}g^{3}-3\,ab^{2}g^{2}h+3\,a^{2}bgh^{2}-a^{3}h^{3}} - \frac{2\,b^{2}\log\left(hx+g\right)}{b^{3}g^{3}-3\,ab^{2}g^{2}h+3\,a^{2}bgh^{2}-a^{3}h^{3}} + \frac{2\,bhx+3\,bg-ah}{b^{2}g^{4}-2\,abg^{3}h+a^{2}g^{2}h^{2}+\left(b^{2}g^{2}h^{2}-2\,abgh^{3}+a^{2}h^{4}\right)x^{2}+2\,\left(b^{2}g^{3}h-2\,abg^{2}h^{2}+a^{2}gh^{3}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*b^2*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3
) - 2*b^2*log(h*x + g)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3)
+ (2*b*h*x + 3*b*g - a*h)/(b^2*g^4 - 2*a*b*g^3*h + a^2*g^2*h^2 + (b^2*g^2*h
^2 - 2*a*b*g*h^3 + a^2*h^4)*x^2 + 2*(b^2*g^3*h - 2*a*b*g^2*h^2 + a^2*g*h^3)
*x))*b*f*p + (2*d^2*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 -
 c^3*h^3) - 2*d^2*log(h*x + g)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c
^3*h^3) + (2*d*h*x + 3*d*g - c*h)/(d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2 + (d
^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*x^2 + 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^
2*g*h^3)*x))*d*f*q)*r/(f*h) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*
x + g)^3*h)
```

mupad [B] time = 5.48, size = 977, normalized size = 3.76

$$\frac{\frac{3\,b^{2}\,d^{2}\,g^{3}\,p\,r+3\,b^{2}\,d^{2}\,g^{3}\,q\,r-a\,b\,c^{2}\,h^{3}\,p\,r-a^{2}\,c\,d\,h^{3}\,q\,r+3\,b^{2}\,c^{2}\,g\,h^{2}\,p\,r+3\,a^{2}\,d^{2}\,g\,h^{2}\,q\,r-a\,b\,d^{2}\,g^{2}\,h\,p\,r-6\,a\,b\,d^{2}\,g^{2}\,h\,q\,r-6\,b^{2}\,c\,d\,g^{2}\,h\,p\,r-b^{2}\,c\,d\,g^{2}\,h\,q\,r+2\,a\,b\,c\,d}{2\left(a^{2}\,c^{2}\,h^{4}-2\,a^{2}\,c\,d\,g\,h^{3}+a^{2}\,d^{2}\,g^{2}\,h^{2}-2\,a\,b\,c^{2}\,g\,h^{3}+4\,a\,b\,c\,d\,g^{2}\,h^{2}-2\,a\,b\,d^{2}\,g^{3}\,h+b^{2}\,c^{2}\,g^{2}\,h^{2}-2\,b^{2}\,c\,d\,g^{3}\,h+b^{2}\,d^{2}\,g^{4}\right)}}{3\,g^{2}\,h+6\,g\,h^{2}\,x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^4,x)
```

```
[Out] ((3*b^2*d^2*g^3*p*r + 3*b^2*d^2*g^3*q*r - a*b*c^2*h^3*p*r - a^2*c*d*h^3*q*r
 + 3*b^2*c^2*g*h^2*p*r + 3*a^2*d^2*g*h^2*q*r - a*b*d^2*g^2*h*p*r - 6*a*b*d^
```

```
2*g^2*h*q*r - 6*b^2*c*d*g^2*h*p*r - b^2*c*d*g^2*h*q*r + 2*a*b*c*d*g*h^2*p*r
 + 2*a*b*c*d*g*h^2*q*r)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b
^2*c^2*g^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^
2*c*d*g^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(b^2*c^2*h^3*p*r + a^2*d^2*h^3*q*r +
 b^2*d^2*g^2*h*p*r + b^2*d^2*g^2*h*q*r - 2*a*b*d^2*g*h^2*q*r - 2*b^2*c*d*g*
h^2*p*r))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 -
2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a
*b*c*d*g^2*h^2))/(3*g^2*h + 3*h^3*x^2 + 6*g*h^2*x) + (log(g + h*x)*(g^2*(3*
a*b^2*d^3*h*q*r + 3*b^3*c*d^2*h*p*r) - g^3*(b^3*d^3*p*r + b^3*d^3*q*r) - g*
(3*a^2*b*d^3*h^2*q*r + 3*b^3*c^2*d*h^2*p*r) + b^3*c^3*h^3*p*r + a^3*d^3*h^3
*q*r))/(3*a^3*c^3*h^7 + 3*b^3*d^3*g^6*h - 3*a^3*d^3*g^3*h^4 - 3*b^3*c^3*g^3
*h^4 - 9*a^2*b*c^3*g*h^6 - 9*a^3*c^2*d*g*h^6 + 9*a*b^2*c^3*g^2*h^5 - 9*a*b^
2*d^3*g^5*h^2 + 9*a^2*b*d^3*g^4*h^3 + 9*a^3*c*d^2*g^2*h^5 - 9*b^3*c*d^2*g^5
*h^2 + 9*b^3*c^2*d*g^4*h^3 + 27*a*b^2*c*d^2*g^4*h^3 - 27*a*b^2*c^2*d*g^3*h^
4 - 27*a^2*b*c*d^2*g^3*h^4 + 27*a^2*b*c^2*d*g^2*h^5) - (log(e*(f*(a + b*x)^
p*(c + d*x)^q)^r)*(x/3 + g/(3*h)))/(g + h*x)^4 - (b^3*p*r*log(a + b*x))/(3*
a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - (d^3*q*r*log(c +
 d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d*g*h^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=318

$$\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}+\frac{b^3pr}{4h(g+hx)(bg-ah)^3}+\frac{b^2pr}{8h(g+hx)^2(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4}$$

```
[Out] 1/12*b*p*r/h/(-a*h+b*g)/(h*x+g)^3+1/12*d*q*r/h/(-c*h+d*g)/(h*x+g)^3+1/8*b^2
*p*r/h/(-a*h+b*g)^2/(h*x+g)^2+1/8*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)^2+1/4*b^3*
p*r/h/(-a*h+b*g)^3/(h*x+g)+1/4*d^3*q*r/h/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*p*r*l
n(b*x+a)/h/(-a*h+b*g)^4+1/4*d^4*q*r*ln(d*x+c)/h/(-c*h+d*g)^4-1/4*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^4-1/4*b^4*p*r*ln(h*x+g)/h/(-a*h+b*g)^4-1/4*
d^4*q*r*ln(h*x+g)/h/(-c*h+d*g)^4
```

Rubi [A] time = 0.19, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^3pr}{4h(g+hx)(bg-ah)^3}+\frac{b^2pr}{8h(g+hx)^2(bg-ah)^2}+\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (b*p*r)/(12*h*(b*g - a*h)*(g + h*x)^3) + (d*q*r)/(12*h*(d*g - c*h)*(g + h*x
)^3) + (b^2*p*r)/(8*h*(b*g - a*h)^2*(g + h*x)^2) + (d^2*q*r)/(8*h*(d*g - c*
h)^2*(g + h*x)^2) + (b^3*p*r)/(4*h*(b*g - a*h)^3*(g + h*x)) + (d^3*q*r)/(4*
h*(d*g - c*h)^3*(g + h*x)) + (b^4*p*r*Log[a + b*x])/(4*h*(b*g - a*h)^4) + (
d^4*q*r*Log[c + d*x])/(4*h*(d*g - c*h)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]/(4*h*(g + h*x)^4) - (b^4*p*r*Log[g + h*x])/(4*h*(b*g - a*h)^4) - (d^4
*q*r*Log[g + h*x])/(4*h*(d*g - c*h)^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^4}\,dx}{4h} + \frac{(dqr)\int \frac{}{(c+dx)}}{4h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int\left(\frac{b^4}{(bg-ah)^4(a+bx)} - \frac{h}{(bg-ah)(g+hx)^4} - \right.}{}$$

$$= \frac{bpr}{12h(bg-ah)(g+hx)^3} + \frac{dqr}{12h(dg-ch)(g+hx)^3} + \frac{b^2pr}{8h(bg-ah)^2(g+hx)^2}$$

Mathematica [A] time = 1.49, size = 480, normalized size = 1.51

$$\frac{r(g+hx)\left(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(adq+bcp))-(g+hx)\left((bg-ah)^2(dg-ch)^2\left(-3a^2d^2h^2q+6abd^2ghq-3b^2\left(c^2h^2p-2cdghp+d^2g^2(p+q)\right)\right)+6(g+hx)\left(-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (-6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*(2*(b*g - a*h)^3*(d
*g - c*h)^3*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)^2*
(d*g - c*h)^2*(6*a*b*d^2*g*h*q - 3*a^2*d^2*h^2*q - 3*b^2*(-2*c*d*g*h*p + c^
2*h^2*p + d^2*g^2*(p + q))) + 6*(g + h*x)*(-((b*g - a*h)*(-(d*g) + c*h)*(3*
a*b^2*d^3*g^2*h*q - 3*a^2*b*d^3*g*h^2*q + a^3*d^3*h^3*q - b^3*(-3*c*d^2*g^2
*h*p + 3*c^2*d*g*h^2*p - c^3*h^3*p + d^3*g^3*(p + q)))) - (g + h*x)*(b^4*(d
*g - c*h)^4*p*Log[a + b*x] + d^4*(b*g - a*h)^4*q*Log[c + d*x] - (-4*a*b^3*d
^4*g^3*h*q + 6*a^2*b^2*d^4*g^2*h^2*q - 4*a^3*b*d^4*g*h^3*q + a^4*d^4*h^4*q
+ b^4*(-4*c*d^3*g^3*h*p + 6*c^2*d^2*g^2*h^2*p - 4*c^3*d*g*h^3*p + c^4*h^4*p
 + d^4*g^4*(p + q)))*Log[g + h*x])))))/((b*g - a*h)^4*(d*g - c*h)^4))/(24*h
*(g + h*x)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.17, size = 3911, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*b^5*p*r*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) + 1/4*d^5*q*r*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*p*r*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
g^4*h) - 1/4*q*r*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^
3*h^2*x + g^4*h) - 1/4*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c^2
*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*q*
r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3*q
*r + a^4*d^4*h^4*q*r)*log(h*x + g)/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^7*h^2 - 4
*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a^2
*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^2*
b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*
g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g^4
*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2*g^
3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2*h^
7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c^4
*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^2*h^4*p*r*x^3 + 18
*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3*d^3*g^3*h^3*q*r*x^
3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*r*x^3 - 6*a^3*d^3*h
^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d^2*g^3*h^3*p*r*x^2 - 3*
a*b^2*d^3*g^3*h^3*p*r*x^2 + 63*b^3*c^2*d*g^2*h^4*p*r*x^2 + 9*a*b^2*c*d^2*g^
2*h^4*p*r*x^2 - 21*b^3*c^3*g*h^5*p*r*x^2 - 9*a*b^2*c^2*d*g*h^5*p*r*x^2 + 3*
a*b^2*c^3*h^6*p*r*x^2 + 21*b^3*d^3*g^4*h^2*q*r*x^2 - 3*b^3*c*d^2*g^3*h^3*q*
r*x^2 - 63*a*b^2*d^3*g^3*h^3*q*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*q*r*x^2 + 63*a
^2*b*d^3*g^2*h^4*q*r*x^2 - 9*a^2*b*c*d^2*g*h^5*q*r*x^2 - 21*a^3*d^3*g*h^5*q
*r*x^2 + 3*a^3*c*d^2*h^6*q*r*x^2 + 26*b^3*d^3*g^5*h*p*r*x - 78*b^3*c*d^2*g^
4*h^2*p*r*x - 10*a*b^2*d^3*g^4*h^2*p*r*x + 78*b^3*c^2*d*g^3*h^3*p*r*x + 30*
a*b^2*c*d^2*g^3*h^3*p*r*x + 2*a^2*b*d^3*g^3*h^3*p*r*x - 26*b^3*c^3*g^2*h^4*
p*r*x - 30*a*b^2*c^2*d*g^2*h^4*p*r*x - 6*a^2*b*c*d^2*g^2*h^4*p*r*x + 10*a*b
^2*c^3*g*h^5*p*r*x + 6*a^2*b*c^2*d*g*h^5*p*r*x - 2*a^2*b*c^3*h^6*p*r*x + 26
*b^3*d^3*g^5*h*q*r*x - 10*b^3*c*d^2*g^4*h^2*q*r*x - 78*a*b^2*d^3*g^4*h^2*q*
r*x + 2*b^3*c^2*d*g^3*h^3*q*r*x + 30*a*b^2*c*d^2*g^3*h^3*q*r*x + 78*a^2*b*d
```

^3*g^3*h^3*q*r*x - 6*a*b^2*c^2*d*g^2*h^4*q*r*x - 30*a^2*b*c*d^2*g^2*h^4*q*r
*x - 26*a^3*d^3*g^2*h^4*q*r*x + 6*a^2*b*c^2*d*g*h^5*q*r*x + 10*a^3*c*d^2*g*
h^5*q*r*x - 2*a^3*c^2*d*h^6*q*r*x + 11*b^3*d^3*g^6*p*r - 33*b^3*c*d^2*g^5*h
*p*r - 7*a*b^2*d^3*g^5*h*p*r + 33*b^3*c^2*d*g^4*h^2*p*r + 21*a*b^2*c*d^2*g^
4*h^2*p*r + 2*a^2*b*d^3*g^4*h^2*p*r - 11*b^3*c^3*g^3*h^3*p*r - 21*a*b^2*c^2
*d*g^3*h^3*p*r - 6*a^2*b*c*d^2*g^3*h^3*p*r + 7*a*b^2*c^3*g^2*h^4*p*r + 6*a^
2*b*c^2*d*g^2*h^4*p*r - 2*a^2*b*c^3*g*h^5*p*r + 11*b^3*d^3*g^6*q*r - 7*b^3*
c*d^2*g^5*h*q*r - 33*a*b^2*d^3*g^5*h*q*r + 2*b^3*c^2*d*g^4*h^2*q*r + 21*a*b
^2*c*d^2*g^4*h^2*q*r + 33*a^2*b*d^3*g^4*h^2*q*r - 6*a*b^2*c^2*d*g^3*h^3*q*r
 - 21*a^2*b*c*d^2*g^3*h^3*q*r - 11*a^3*d^3*g^3*h^3*q*r + 6*a^2*b*c^2*d*g^2*
h^4*q*r + 7*a^3*c*d^2*g^2*h^4*q*r - 2*a^3*c^2*d*g*h^5*q*r - 6*b^3*d^3*g^6*r
*log(f) + 18*b^3*c*d^2*g^5*h*r*log(f) + 18*a*b^2*d^3*g^5*h*r*log(f) - 18*b^
3*c^2*d*g^4*h^2*r*log(f) - 54*a*b^2*c*d^2*g^4*h^2*r*log(f) - 18*a^2*b*d^3*g
^4*h^2*r*log(f) + 6*b^3*c^3*g^3*h^3*r*log(f) + 54*a*b^2*c^2*d*g^3*h^3*r*log
(f) + 54*a^2*b*c*d^2*g^3*h^3*r*log(f) + 6*a^3*d^3*g^3*h^3*r*log(f) - 18*a*b
^2*c^3*g^2*h^4*r*log(f) - 54*a^2*b*c^2*d*g^2*h^4*r*log(f) - 18*a^3*c*d^2*g^
2*h^4*r*log(f) + 18*a^2*b*c^3*g*h^5*r*log(f) + 18*a^3*c^2*d*g*h^5*r*log(f)
 - 6*a^3*c^3*h^6*r*log(f) - 6*b^3*d^3*g^6 + 18*b^3*c*d^2*g^5*h + 18*a*b^2*d^
3*g^5*h - 18*b^3*c^2*d*g^4*h^2 - 54*a*b^2*c*d^2*g^4*h^2 - 18*a^2*b*d^3*g^4*
h^2 + 6*b^3*c^3*g^3*h^3 + 54*a*b^2*c^2*d*g^3*h^3 + 54*a^2*b*c*d^2*g^3*h^3 +
 6*a^3*d^3*g^3*h^3 - 18*a*b^2*c^3*g^2*h^4 - 54*a^2*b*c^2*d*g^2*h^4 - 18*a^3
*c*d^2*g^2*h^4 + 18*a^2*b*c^3*g*h^5 + 18*a^3*c^2*d*g*h^5 - 6*a^3*c^3*h^6)/(
b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 - 3*a*b^2*d^3*g^5*h^6*x^4 + 3
*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*x^4 + 3*a^2*b*d^3*g^4*h^7*x^
4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8*x^4 - 9*a^2*b*c*d^2*g^3*h^8
*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^9*x^4 + 9*a^2*b*c^2*d*g^2*h^
9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g*h^10*x^4 - 3*a^3*c^2*d*g*h^
10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x^3 - 12*b^3*c*d^2*g^6*h^5*x^
3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5*h^6*x^3 + 36*a*b^2*c*d^2*g^
5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^3*g^4*h^7*x^3 - 36*a*b^2*c^2
*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4*a^3*d^3*g^4*h^7*x^3 + 12*a*
b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^3 + 12*a^3*c*d^2*g^3*h^8*x^3
 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*h^9*x^3 + 4*a^3*c^3*g*h^10*x
^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h^4*x^2 - 18*a*b^2*d^3*g^7*h^
4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d^2*g^6*h^5*x^2 + 18*a^2*b*d^
3*g^6*h^5*x^2 - 6*b^3*c^3*g^5*h^6*x^2 - 54*a*b^2*c^2*d*g^5*h^6*x^2 - 54*a^2
*b*c*d^2*g^5*h^6*x^2 - 6*a^3*d^3*g^5*h^6*x^2 + 18*a*b^2*c^3*g^4*h^7*x^2 + 5
4*a^2*b*c^2*d*g^4*h^7*x^2 + 18*a^3*c*d^2*g^4*h^7*x^2 - 18*a^2*b*c^3*g^3*h^8
*x^2 - 18*a^3*c^2*d*g^3*h^8*x^2 + 6*a^3*c^3*g^2*h^9*x^2 + 4*b^3*d^3*g^9*h^2
*x - 12*b^3*c*d^2*g^8*h^3*x - 12*a*b^2*d^3*g^8*h^3*x + 12*b^3*c^2*d*g^7*h^4
*x + 36*a*b^2*c*d^2*g^7*h^4*x + 12*a^2*b*d^3*g^7*h^4*x - 4*b^3*c^3*g^6*h^5*
x - 36*a*b^2*c^2*d*g^6*h^5*x - 36*a^2*b*c*d^2*g^6*h^5*x - 4*a^3*d^3*g^6*h^5
*x + 12*a*b^2*c^3*g^5*h^6*x + 36*a^2*b*c^2*d*g^5*h^6*x + 12*a^3*c*d^2*g^5*h
^6*x - 12*a^2*b*c^3*g^4*h^7*x - 12*a^3*c^2*d*g^4*h^7*x + 4*a^3*c^3*g^3*h^8*
x + b^3*d^3*g^10*h - 3*b^3*c*d^2*g^9*h^2 - 3*a*b^2*d^3*g^9*h^2 + 3*b^3*c^2*

```
d*g^8*h^3 + 9*a*b^2*c*d^2*g^8*h^3 + 3*a^2*b*d^3*g^8*h^3 - b^3*c^3*g^7*h^4 -
 9*a*b^2*c^2*d*g^7*h^4 - 9*a^2*b*c*d^2*g^7*h^4 - a^3*d^3*g^7*h^4 + 3*a*b^2*
c^3*g^6*h^5 + 9*a^2*b*c^2*d*g^6*h^5 + 3*a^3*c*d^2*g^6*h^5 - 3*a^2*b*c^3*g^5
*h^6 - 3*a^3*c^2*d*g^5*h^6 + a^3*c^3*g^4*h^7)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

maxima [B] time = 1.03, size = 776, normalized size = 2.44

$$\left(\left(\frac{6\,b^3\log(bx+a)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} - \frac{6\,b^3\log\left(hx+g\right)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} + \frac{}{b^3g^6-3\,ab^2g^5h+3\,a^2bg^4h^2-a^3g^3h^3+\left(b^3g^3h^3-3\,ab^2g^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 1/24*((6*b^3*log(b*x + a)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^2*b^2*g^2*h^2 - 4*
a^3*b*g*h^3 + a^4*h^4) - 6*b^3*log(h*x + g)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^
2*b^2*g^2*h^2 - 4*a^3*b*g*h^3 + a^4*h^4) + (6*b^2*h^2*x^2 + 11*b^2*g^2 - 7*
a*b*g*h + 2*a^2*h^2 + 3*(5*b^2*g*h - a*b*h^2)*x)/(b^3*g^6 - 3*a*b^2*g^5*h +
 3*a^2*b*g^4*h^2 - a^3*g^3*h^3 + (b^3*g^3*h^3 - 3*a*b^2*g^2*h^4 + 3*a^2*b*g
*h^5 - a^3*h^6)*x^3 + 3*(b^3*g^4*h^2 - 3*a*b^2*g^3*h^3 + 3*a^2*b*g^2*h^4 -
a^3*g*h^5)*x^2 + 3*(b^3*g^5*h - 3*a*b^2*g^4*h^2 + 3*a^2*b*g^3*h^3 - a^3*g^2
*h^4)*x))*b*f*p + (6*d^3*log(d*x + c)/(d^4*g^4 - 4*c*d^3*g^3*h + 6*c^2*d^2*
g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) - 6*d^3*log(h*x + g)/(d^4*g^4 - 4*c*d^3*
g^3*h + 6*c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) + (6*d^2*h^2*x^2 + 11*
d^2*g^2 - 7*c*d*g*h + 2*c^2*h^2 + 3*(5*d^2*g*h - c*d*h^2)*x)/(d^3*g^6 - 3*c
*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3 + (d^3*g^3*h^3 - 3*c*d^2*g^2*h^4
 + 3*c^2*d*g*h^5 - c^3*h^6)*x^3 + 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*
d*g^2*h^4 - c^3*g*h^5)*x^2 + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h
^3 - c^3*g^2*h^4)*x))*d*f*q)*r/(f*h) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/((h*x + g)^4*h)
```

mupad [B] time = 10.15, size = 2215, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^5,x)
```

```
[Out] ((11*b^3*d^3*g^5*p*r + 11*b^3*d^3*g^5*q*r - 11*b^3*c^3*g^2*h^3*p*r - 11*a^3
*d^3*g^2*h^3*q*r - 2*a^2*b*c^3*h^5*p*r - 2*a^3*c^2*d*h^5*q*r + 7*a*b^2*c^3*
g*h^4*p*r - 7*a*b^2*d^3*g^4*h*p*r - 33*a*b^2*d^3*g^4*h*q*r + 7*a^3*c*d^2*g*
h^4*q*r - 33*b^3*c*d^2*g^4*h*p*r - 7*b^3*c*d^2*g^4*h*q*r + 2*a^2*b*d^3*g^3*
h^2*p*r + 33*a^2*b*d^3*g^3*h^2*q*r + 33*b^3*c^2*d*g^3*h^2*p*r + 2*b^3*c^2*d
*g^3*h^2*q*r + 21*a*b^2*c*d^2*g^3*h^2*p*r - 21*a*b^2*c^2*d*g^2*h^3*p*r - 6*
a^2*b*c*d^2*g^2*h^3*p*r + 21*a*b^2*c*d^2*g^3*h^2*q*r - 6*a*b^2*c^2*d*g^2*h^
3*q*r - 21*a^2*b*c*d^2*g^2*h^3*q*r + 6*a^2*b*c^2*d*g*h^4*p*r + 6*a^2*b*c^2*
d*g*h^4*q*r)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*
h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d
^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4
+ 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a
^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (x^2*(b^3*c^3*h^5*p*r + a^3*
d^3*h^5*q*r - b^3*d^3*g^3*h^2*p*r - b^3*d^3*g^3*h^2*q*r - 3*a^2*b*d^3*g*h^4
*q*r - 3*b^3*c^2*d*g*h^4*p*r + 3*a*b^2*d^3*g^2*h^3*q*r + 3*b^3*c*d^2*g^2*h^
3*p*r))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*
a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h
 + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*
c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d
^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x*(a*b^2*c^3*h^5*p*r + a^3*c*d^2*h^5
*q*r - 5*b^3*c^3*g*h^4*p*r - 5*a^3*d^3*g*h^4*q*r + 5*b^3*d^3*g^4*h*p*r + 5*
b^3*d^3*g^4*h*q*r - a*b^2*d^3*g^3*h^2*p*r - 15*a*b^2*d^3*g^3*h^2*q*r + 15*a
^2*b*d^3*g^2*h^3*q*r - 15*b^3*c*d^2*g^3*h^2*p*r + 15*b^3*c^2*d*g^2*h^3*p*r
- b^3*c*d^2*g^3*h^2*q*r + 3*a*b^2*c*d^2*g^2*h^3*p*r + 3*a*b^2*c*d^2*g^2*h^3
*q*r - 3*a*b^2*c^2*d*g*h^4*p*r - 3*a^2*b*c*d^2*g*h^4*q*r))/(2*(a^3*c^3*h^6
+ b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a
*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^
4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b
^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*
c^2*d*g^2*h^4)))/(4*g^3*h + 4*h^4*x^3 + 12*g^2*h^2*x + 12*g*h^3*x^2) - (log
(g + h*x)*(h^4*(b^4*c^4*p*r + a^4*d^4*q*r) - h*(4*a*b^3*d^4*g^3*q*r + 4*b^4
*c*d^3*g^3*p*r) + h^2*(6*a^2*b^2*d^4*g^2*q*r + 6*b^4*c^2*d^2*g^2*p*r) - h^3
*(4*a^3*b*d^4*g*q*r + 4*b^4*c^3*d*g*p*r) + b^4*d^4*g^4*p*r + b^4*d^4*g^4*q*
r))/(4*a^4*c^4*h^9 + 4*b^4*d^4*g^8*h + 4*a^4*d^4*g^4*h^5 + 4*b^4*c^4*g^4*h^
5 + 24*a^2*b^2*c^4*g^2*h^7 + 24*a^2*b^2*d^4*g^6*h^3 + 24*a^4*c^2*d^2*g^2*h^
7 + 24*b^4*c^2*d^2*g^6*h^3 - 16*a^3*b*c^4*g*h^8 - 16*a^4*c^3*d*g*h^8 - 16*a
*b^3*c^4*g^3*h^6 - 16*a*b^3*d^4*g^7*h^2 - 16*a^3*b*d^4*g^5*h^4 - 16*a^4*c*d
^3*g^3*h^6 - 16*b^4*c*d^3*g^7*h^2 - 16*b^4*c^3*d*g^5*h^4 + 64*a*b^3*c*d^3*g
^6*h^3 + 64*a*b^3*c^3*d*g^4*h^5 + 64*a^3*b*c*d^3*g^4*h^5 + 64*a^3*b*c^3*d*g
^2*h^7 - 96*a*b^3*c^2*d^2*g^5*h^4 - 96*a^2*b^2*c*d^3*g^5*h^4 - 96*a^2*b^2*c
^3*d*g^3*h^6 - 96*a^3*b*c^2*d^2*g^3*h^6 + 144*a^2*b^2*c^2*d^2*g^4*h^5) - (l
```

```
og(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + g/(4*h)))/(g + h*x)^5 + (b^4*p*r
*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a^2*b^2*g^2
*h^3 - 16*a^3*b*g*h^4) + (d^4*q*r*log(c + d*x))/(4*c^4*h^5 + 4*d^4*g^4*h -
16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**5,x)
[Out] Timed out
```

3.35 $\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=2240

$$-\frac{p^2r^2\log^2(a+bx)(bg-ah)^4}{4b^4h}+\frac{pqr^2\log(a+bx)(bg-ah)^4}{8b^4h}-\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^4}{2b^4h}+\frac{pr\log(a}{}$$

```
[Out] 1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+1/8*(-a*h+b*g)^4*p*q*r^2*
ln(b*x+a)/b^4/h-1/2*(-c*h+d*g)^3*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^3-1/4*(-c*h+
d*g)^2*p*q*r^2*(h*x+g)^2*ln(b*x+a)/d^2/h-1/6*(-c*h+d*g)*p*q*r^2*(h*x+g)^3*l
n(b*x+a)/d/h-1/2*(-a*h+b*g)^3*p*q*r^2*(d*x+c)*ln(d*x+c)/b^3/d-1/4*(-a*h+b*g
)^2*p*q*r^2*(h*x+g)^2*ln(d*x+c)/b^2/h-1/6*(-a*h+b*g)*p*q*r^2*(h*x+g)^3*ln(d
*x+c)/b/h-1/2*(-a*h+b*g)^4*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^4/
h-1/2*(-c*h+d*g)^4*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^4/h+1/6*(-a
*h+b*g)^3*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^3/d/h+1/4*(-a*h+b*g)^2*(-c*h+d*g)^
2*p*q*r^2*ln(b*x+a)/b^2/d^2/h+1/4*(-a*h+b*g)^2*(-c*h+d*g)^2*p*q*r^2*ln(d*x+
c)/b^2/d^2/h+1/6*(-a*h+b*g)*(-c*h+d*g)^3*p*q*r^2*ln(d*x+c)/b/d^3/h+1/6*(-a*
h+b*g)*(-c*h+d*g)*p*q*r^2*(h*x+g)^2/b/d/h-3/2*h*(-a*h+b*g)^2*p^2*r^2*(b*x+a
)^2*ln(b*x+a)/b^4-2/3*h^2*(-a*h+b*g)*p^2*r^2*(b*x+a)^3*ln(b*x+a)/b^4+1/8*(-
c*h+d*g)^4*p*q*r^2*ln(d*x+c)/d^4/h-3/2*h*(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2*ln(
d*x+c)/d^4-2/3*h^2*(-c*h+d*g)*q^2*r^2*(d*x+c)^3*ln(d*x+c)/d^4+1/4*(-a*h+b*g
)^2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r))/b^2/h+1/4*(-c*h+d*g)^2*q*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e
*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h+1/6*(-a*h+b*g)*p*r*(h*x+g)^3*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b/h+1/6*(-c*h+d*g)*q*r*(
h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d/h+
1/2*(-a*h+b*g)^4*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r))/b^4/h+1/2*(-c*h+d*g)^4*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*l
n(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^4/h+5/12*(-a*h+b*g)^2*(-c*h+d*g
)*p*q*r^2*x/b^2/d+5/12*(-a*h+b*g)*(-c*h+d*g)^2*p*q*r^2*x/b/d^2+1/8*p*r*(h*x
+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+1/8*q
*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/
h+1/2*(-c*h+d*g)^3*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r))/d^3-2*(-a*h+b*g)^3*p^2*r^2*(b*x+a)*ln(b*x+a)/b^4-1/8*h^3*p^2*r^
2*(b*x+a)^4*ln(b*x+a)/b^4-1/8*p*q*r^2*(h*x+g)^4*ln(b*x+a)/h-1/4*(-a*h+b*g)^
4*p^2*r^2*ln(b*x+a)^2/b^4/h-2*(-c*h+d*g)^3*q^2*r^2*(d*x+c)*ln(d*x+c)/d^4-1/
8*h^3*q^2*r^2*(d*x+c)^4*ln(d*x+c)/d^4-1/8*p*q*r^2*(h*x+g)^4*ln(d*x+c)/h-1/4
*(-c*h+d*g)^4*q^2*r^2*ln(d*x+c)^2/d^4/h+1/2*(-a*h+b*g)^3*p*r*x*(p*r*ln(b*x+
a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^3+3/16*(-a*h+b*g)^2*p*q
*r^2*(h*x+g)^2/b^2/h+3/16*(-c*h+d*g)^2*p*q*r^2*(h*x+g)^2/d^2/h+7/72*(-a*h+b
*g)*p*q*r^2*(h*x+g)^3/b/h+7/72*(-c*h+d*g)*p*q*r^2*(h*x+g)^3/d/h-1/2*(-c*h+d
*g)^4*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^4/h-1/2*(-a*h+b*g)^4*p*q*r
^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b^4/h+2*(-a*h+b*g)^3*p^2*r^2*x/b^3+2*(-c
```

```
*h+d*g)^3*q^2*r^2*x/d^3+1/32*h^3*p^2*r^2*(b*x+a)^4/b^4+1/32*h^3*q^2*r^2*(d*
x+c)^4/d^4+1/16*p*q*r^2*(h*x+g)^4/h+5/8*(-a*h+b*g)^3*p*q*r^2*x/b^3+5/8*(-c*
h+d*g)^3*p*q*r^2*x/d^3+3/4*h*(-a*h+b*g)^2*p^2*r^2*(b*x+a)^2/b^4+2/9*h^2*(-a
*h+b*g)*p^2*r^2*(b*x+a)^3/b^4+3/4*h*(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2/d^4+2/9*
h^2*(-c*h+d*g)*q^2*r^2*(d*x+c)^3/d^4
```

Rubi [A] time = 2.44, antiderivative size = 2220, normalized size of antiderivative = 0.99, number of steps used = 49, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2513, 2411, 43, 2334, 12, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]

[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
 c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)
/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
 c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
 b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - ((d*g - c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - ((d
*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)*p*q*
r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b*x])/
(8*h) + ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) - (p^2*r^2*Log[a +
 b*x]*((48*h*(b*g - a*h)^3*(a + b*x))/b^4 + (36*h^2*(b*g - a*h)^2*(a + b*x)
^2)/b^4 + (16*h^3*(b*g - a*h)*(a + b*x)^3)/b^4 + (3*h^4*(a + b*x)^4)/b^4 +
(12*(b*g - a*h)^4*Log[a + b*x])/b^4))/(24*h) + ((b*g - a*h)^2*(d*g - c*h)^2
*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g - c*h)^3*p*q*r^2*L
og[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c + d*x])/(8*d^4*h) -
 ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d) - ((b*g - a*h)^2*
p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g - a*h)*p*q*r^2*(g + h*x
)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[c + d*x])/(8*h) - ((b*
g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b^4*h
) + ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h) - (q^2*r^2*Log[c + d*x
]*((48*h*(d*g - c*h)^3*(c + d*x))/d^4 + (36*h^2*(d*g - c*h)^2*(c + d*x)^2)/
d^4 + (16*h^3*(d*g - c*h)*(c + d*x)^3)/d^4 + (3*h^4*(c + d*x)^4)/d^4 + (12*
(d*g - c*h)^4*Log[c + d*x])/d^4))/(24*h) - ((d*g - c*h)^4*p*q*r^2*Log[a + b
```

```
*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^4*h) + ((b*g - a*h)^3*p*r*x*(p*r*L
og[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*
b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^3) + ((b*g - a*h)^2*p*r*(g + h*x)^2*(p*
r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/
(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(4*d^2*h) + ((b*g - a*h)*p*r*
(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(6*b*h) + ((d*g - c*h)*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q
*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(6*d*h) + (p*r*(g
+ h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(8*h) + (q*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(8*h) + ((b*g - a*h)^4*p*r*Log[a +
b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r
*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^4*h) + ((g + h*
x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2
*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*d^4*h) - ((b*g - a*h)^4*p*q*r
^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b^4*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4 \log\left(e\left(f(\right.\right.}{a}}{2h}$$

$$= \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{\left(bp^2r^2\right)\int \frac{(g+hx)^4 \log(a}{a+bx}}{2h}$$

$$= \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\left(\frac{bg-}{b}\right.}{}\right.}{}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{48h(bg-ah)^3(a+bx)}{b^4} + \frac{36h^2(bg-ah)^2(a+bx)^2}{b^4} + \frac{16h^3(bg-}{}\right.}{24h}$$

$$= -\frac{(dg-ch)^2pqr^2(g+hx)^2\log(a+bx)}{4d^2h} - \frac{(dg-ch)pqr^2(g+hx)^3}{6dh}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{(bg-ah)^3pqr^2x}{2b^3} + \frac{(dg-ch)^3pqr^2x}{2d^3} + \frac{2(dg}{}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{5(bg-ah)^3pqr^2x}{8b^3} + \frac{5(bg-ah)^2(dg-ch)pq}{12b^2d}$$

Mathematica [A] time = 3.39, size = 1386, normalized size = 0.62

$$\frac{72a\left(-4b^3g^3+6ab^2hg^2-4a^2bh^2g+a^3h^3\right)p^2r^2\log^2(a+bx)d^4+12pr\log(a+bx)\left(12c\left(-4d^3g^3+6cd^2hg^2-4c^2d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (72*a*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*p^2*r^2*Lo
g[a + b*x]^2 + 12*p*r*Log[a + b*x]*(12*b^4*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h -
4*c^2*d*g*h^2 + c^3*h^3)*q*r*Log[c + d*x] - 12*(4*a*b^3*d^4*g^3 - 6*a^2*b^2
*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d^3*g^3 + 6*c*d^2*
g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*
((12*b^3*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*q + a^3*d^3
*h^3*(25*p + 3*q) - 4*a^2*b*d^2*h^2*(22*d*g*p + 4*d*g*q - c*h*q) + 6*a*b^2*
d*h*(-4*c*d*g*h*q + c^2*h^2*q + 6*d^2*g^2*(3*p + q)))*r + 12*d^3*(4*b^3*g^3
```

```
 - 6*a*b^2*g^2*h + 4*a^2*b*g*h^2 - a^3*h^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])) + b*(72*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3
)*q^2*r^2*Log[c + d*x]^2 + 12*q*r*Log[c + d*x]*((12*a^3*c*d^3*h^3*p + 6*a^2
*b*c*d^2*h^2*(-8*d*g + c*h)*p + 4*a*b^2*d*(12*d^3*g^3 + 18*c*d^2*g^2*h - 6*
c^2*d*g*h^2 + c^3*h^3)*p + b^3*c*(-48*d^3*g^3*(p + q) + 36*c*d^2*g^2*h*(p +
 3*q) - 8*c^2*d*g*h^2*(2*p + 11*q) + c^3*h^3*(3*p + 25*q)))*r - 12*b^3*c*(-
4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]) + d*(r^2*(-60*a^3*d^3*h^3*p*(5*p + 3*q)*x + 6*a^2*b*d^2*h^2
*p*x*(-20*c*h*q + 16*d*g*(11*p + 8*q) + d*h*(13*p + 9*q)*x) + b^3*x*(-60*c^
3*h^3*q*(3*p + 5*q) + 6*c^2*d*h^2*q*(16*g*(8*p + 11*q) + h*(9*p + 13*q)*x)
- 4*c*d^2*h*q*(p + q)*(324*g^2 + 60*g*h*x + 7*h^2*x^2) + d^3*(p + q)^2*(576
*g^3 + 216*g^2*h*x + 64*g*h^2*x^2 + 9*h^3*x^3)) - 4*a*b^2*p*(36*c^3*h^3*q +
 6*c^2*d*h^2*q*(-24*g + 5*h*x) - 12*c*d^2*h*q*(-18*g^2 + 12*g*h*x + h^2*x^2
) + d^3*(-144*g^3*q + 324*g^2*h*(p + q)*x + 60*g*h^2*(p + q)*x^2 + 7*h^3*(p
 + q)*x^3))) + 12*r*(12*a^3*d^3*h^3*p*x - 6*a^2*b*d^3*h^2*p*x*(8*g + h*x) +
 4*a*b^2*d^3*p*(-12*g^3 + 18*g^2*h*x + 6*g*h^2*x^2 + h^3*x^3) - b^3*x*(-12*
c^3*h^3*q + 6*c^2*d*h^2*q*(8*g + h*x) - 4*c*d^2*h*q*(18*g^2 + 6*g*h*x + h^2
*x^2) + d^3*(p + q)*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3)))*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^3*d^3*x*(4*g^3 + 6*g^2*h*x + 4*g*h^
2*x^2 + h^3*x^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) - 144*(4*a*b^3*d^
4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d
^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*p*q*r^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(288*b^4*d^4)
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(h^3x^3+3\,gh^2x^2+3\,g^2hx+g^3\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{3} \ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.31, size = 1799, normalized size = 0.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3
*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*
d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*
d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d
^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^
3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b
^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(
p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*
d^3))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*(12*a^3*c*d^3*
f^2*h^3*p*q - 6*(8*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q)*a^2*b + 4*(18
*c*d^3*f^2*g^2*h*p*q - 6*c^2*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q)*a*b^2 -
 (48*(p*q + q^2)*c*d^3*f^2*g^3 - 36*(p*q + 3*q^2)*c^2*d^2*f^2*g^2*h + 8*(2*
p*q + 11*q^2)*c^3*d*f^2*g*h^2 - (3*p*q + 25*q^2)*c^4*f^2*h^3)*b^3)*log(d*x
+ c)/(b^3*d^4) - 144*(4*a*b^3*d^4*f^2*g^3*p*q - 6*a^2*b^2*d^4*f^2*g^2*h*p*q
 + 4*a^3*b*d^4*f^2*g*h^2*p*q - a^4*d^4*f^2*h^3*p*q - (4*c*d^3*f^2*g^3*p*q -
 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2*h^3*p*q)*b^4)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b^4*d^4) + (9*(p^2 + 2*p*q + q^2)*b^4*d^4*f^2*h^3*x^4 - 144*(4*c*
d^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2
*h^3*p*q)*b^4*log(b*x + a)*log(d*x + c) - 72*(4*c*d^3*f^2*g^3*q^2 - 6*c^2*d
^2*f^2*g^2*h*q^2 + 4*c^3*d*f^2*g*h^2*q^2 - c^4*f^2*h^3*q^2)*b^4*log(d*x + c
)^2 - 4*(7*(p^2 + p*q)*a*b^3*d^4*f^2*h^3 - (16*(p^2 + 2*p*q + q^2)*d^4*f^2*
g*h^2 - 7*(p*q + q^2)*c*d^3*f^2*h^3)*b^4)*x^3 + 6*((13*p^2 + 9*p*q)*a^2*b^2
*d^4*f^2*h^3 + 8*(c*d^3*f^2*h^3*p*q - 5*(p^2 + p*q)*d^4*f^2*g*h^2)*a*b^3 +
(36*(p^2 + 2*p*q + q^2)*d^4*f^2*g^2*h - 40*(p*q + q^2)*c*d^3*f^2*g*h^2 + (9
```

```
*p*q + 13*q^2)*c^2*d^2*f^2*h^3)*b^4)*x^2 - 72*(4*a*b^3*d^4*f^2*g^3*p^2 - 6*
a^2*b^2*d^4*f^2*g^2*h*p^2 + 4*a^3*b*d^4*f^2*g*h^2*p^2 - a^4*d^4*f^2*h^3*p^2
)*log(b*x + a)^2 - 12*(5*(5*p^2 + 3*p*q)*a^3*b*d^4*f^2*h^3 + 2*(5*c*d^3*f^2
*h^3*p*q - 4*(11*p^2 + 8*p*q)*d^4*f^2*g*h^2)*a^2*b^2 - 2*(24*c*d^3*f^2*g*h^
2*p*q - 5*c^2*d^2*f^2*h^3*p*q - 54*(p^2 + p*q)*d^4*f^2*g^2*h)*a*b^3 - (48*(
p^2 + 2*p*q + q^2)*d^4*f^2*g^3 - 108*(p*q + q^2)*c*d^3*f^2*g^2*h + 8*(8*p*q
 + 11*q^2)*c^2*d^2*f^2*g*h^2 - 5*(3*p*q + 5*q^2)*c^3*d*f^2*h^3)*b^4)*x + 12
*((25*p^2 + 3*p*q)*a^4*d^4*f^2*h^3 + 4*(c*d^3*f^2*h^3*p*q - 2*(11*p^2 + 2*p
*q)*d^4*f^2*g*h^2)*a^3*b - 6*(4*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q -
 6*(3*p^2 + p*q)*d^4*f^2*g^2*h)*a^2*b^2 + 12*(6*c*d^3*f^2*g^2*h*p*q - 4*c^2
*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q - 4*(p^2 + p*q)*d^4*f^2*g^3)*a*b^3)*
log(b*x + a))/(b^4*d^4))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\left(g+h\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Timed out
```

3.36 $\int (g+hx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=1645

$$-\frac{p^2r^2\log^2(a+bx)(bg-ah)^3}{3b^3h}+\frac{2pqr^2\log(a+bx)(bg-ah)^3}{9b^3h}-\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^3}{3b^3h}+\frac{2pr\log(}{}$$

```
[Out] 1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+8/9*(-a*h+b*g)^2*p*q*r^2*
x/b^2+8/9*(-c*h+d*g)^2*p*q*r^2*x/d^2+1/2*h*(-a*h+b*g)*p^2*r^2*(b*x+a)^2/b^3
+1/2*h*(-c*h+d*g)*q^2*r^2*(d*x+c)^2/d^3-2/3*(-c*h+d*g)^2*p*q*r^2*(b*x+a)*ln
(b*x+a)/b/d^2-1/3*(-c*h+d*g)*p*q*r^2*(h*x+g)^2*ln(b*x+a)/d/h-2/3*(-a*h+b*g)
^2*p*q*r^2*(d*x+c)*ln(d*x+c)/b^2/d-1/3*(-a*h+b*g)*p*q*r^2*(h*x+g)^2*ln(d*x+
c)/b/h-2/3*(-a*h+b*g)^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^3/h-2
/3*(-c*h+d*g)^3*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^3/h+1/3*(-a*h+
b*g)^2*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^2/d/h+1/3*(-a*h+b*g)*(-c*h+d*g)^2*p*q
*r^2*ln(d*x+c)/b/d^2/h-h*(-a*h+b*g)*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^3-h*(-c*h
+d*g)*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^3+2/9*(-a*h+b*g)^3*p*q*r^2*ln(b*x+a)/b^
3/h+2/9*(-c*h+d*g)^3*p*q*r^2*ln(d*x+c)/d^3/h+1/3*(-a*h+b*g)*p*r*(h*x+g)^2*(
p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b/h+1/3*(-c*h+
d*g)*q*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r))/d/h+2/3*(-a*h+b*g)^3*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r))/b^3/h+2/3*(-c*h+d*g)^3*q*r*ln(d*x+c)*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^3/h+2/3*(-a*h+b*g)*(-
c*h+d*g)*p*q*r^2*x/b/d+2/9*p*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r))/h+2/9*q*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c
)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h-2*(-a*h+b*g)^2*p^2*r^2*(b*x+a)*ln(b*x+
a)/b^3-2/9*h^2*p^2*r^2*(b*x+a)^3*ln(b*x+a)/b^3-2/9*p*q*r^2*(h*x+g)^3*ln(b*x
+a)/h-1/3*(-a*h+b*g)^3*p^2*r^2*ln(b*x+a)^2/b^3/h-2*(-c*h+d*g)^2*q^2*r^2*(d*
x+c)*ln(d*x+c)/d^3-2/9*h^2*q^2*r^2*(d*x+c)^3*ln(d*x+c)/d^3-2/9*p*q*r^2*(h*x
+g)^3*ln(d*x+c)/h-1/3*(-c*h+d*g)^3*q^2*r^2*ln(d*x+c)^2/d^3/h+2/3*(-a*h+b*g)
^2*p*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2+
2/3*(-c*h+d*g)^2*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+
c)^q)^r))/d^2+5/18*(-a*h+b*g)*p*q*r^2*(h*x+g)^2/b/h+5/18*(-c*h+d*g)*p*q*r^2
*(h*x+g)^2/d/h-2/3*(-c*h+d*g)^3*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^
3/h-2/3*(-a*h+b*g)^3*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b^3/h+2*(-a*h+
b*g)^2*p^2*r^2*x/b^2+2*(-c*h+d*g)^2*q^2*r^2*x/d^2+2/27*h^2*p^2*r^2*(b*x+a)^
3/b^3+2/27*h^2*q^2*r^2*(d*x+c)^3/d^3+4/27*p*q*r^2*(h*x+g)^3/h
```

Rubi [A] time = 1.76, antiderivative size = 1657, normalized size of antiderivative = 1.01, number of steps used = 47, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*
(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9
*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^
2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*
(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h
)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d
*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x
])/(9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h)
 - (2*(d*g - c*h)^2*p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - ((d*g - c*h
)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*x)^3*Log[a
+ b*x])/(9*h) + ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h) - (p^2*r^2
*Log[a + b*x]*((18*h*(b*g - a*h)^2*(a + b*x))/b^3 + (9*h^2*(b*g - a*h)*(a +
 b*x)^2)/b^3 + (2*h^3*(a + b*x)^3)/b^3 + (6*(b*g - a*h)^3*Log[a + b*x])/b^3
))/(9*h) + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (
2*(d*g - c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*
(c + d*x)*Log[c + d*x])/(3*b^2*d) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log[c
+ d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g - a*
h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) + ((
d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (q^2*r^2*Log[c + d*x]*((18
*h*(d*g - c*h)^2*(c + d*x))/d^3 + (9*h^2*(d*g - c*h)*(c + d*x)^2)/d^3 + (2*
h^3*(c + d*x)^3)/d^3 + (6*(d*g - c*h)^3*Log[c + d*x])/d^3))/(9*h) - (2*(d*g
 - c*h)^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d^3*h) +
(2*(b*g - a*h)^2*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x
] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^2) + ((b
*g - a*h)*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r]))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Lo
g[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d
*h) + (2*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] +
q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*(b*g -
 a*h)^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(
p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
)/(3*d^3*h) + ((g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) -
(2*(d*g - c*h)^3*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h
) - (2*(b*g - a*h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b^3*
h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(2bpr)\int \frac{(g+hx)^3\log\left(e(\ldots\right.}{\ldots}}{3h\ldots}$$

$$= \frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{\left(2bp^2r^2\right)\int \frac{(g+hx)^3\log\ldots}{a+bx\ldots}}{3h}$$

$$= \frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{\left(2p^2r^2\right)\mathrm{Subst}\left(\int \frac{\left(bg\ldots\right.}{\ldots}\right.}{\ldots}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{18h(bg-ah)^2(a+bx)}{b^3}+\frac{9h^2(bg-ah)(a+bx)^2}{b^3}+\frac{2h^3(a+bx)^3}{b^3}\ldots\right.}{9h}$$

$$= -\frac{(dg-ch)pqr^2(g+hx)^2\log(a+bx)}{3dh} - \frac{2pqr^2(g+hx)^3\log(a+b\ldots}{9h}$$

$$= \frac{2(bg-ah)^2pqr^2x}{3b^2} + \frac{2(dg-ch)^2pqr^2x}{3d^2} - \frac{2(dg-ch)^2pqr^2(a+bx\ldots}{3bd^2}$$

$$= \frac{8(bg-ah)^2pqr^2x}{9b^2} + \frac{2(bg-ah)(dg-ch)pqr^2x}{3bd} + \frac{8(dg-ch)^2pqr^2\ldots}{9d^2}$$

$$= \frac{2(bg-ah)^2p^2r^2x}{b^2} + \frac{8(bg-ah)^2pqr^2x}{9b^2} + \frac{2(bg-ah)(dg-ch)pqr^2\ldots}{3bd}$$

Mathematica [A] time = 2.03, size = 899, normalized size = 0.55

$$\frac{-18a\left(3b^2g^2-3abhg+a^2h^2\right)p^2r^2\log^2(a+bx)d^3-6pr\log(a+bx)\left(6c\left(3d^2g^2-3cdhg+c^2h^2\right)qr\log(c+dx)b^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-18*a*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*p^2*r^2*Log[a + b*x]^2 - 6*p*r
*Log[a + b*x]*(6*b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q*r*Log[c + d*x] -
 6*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*((6*b^2*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*q + a^2*d^2*h^2*(11*p + 2*q) - 3*a*b*d*h*(-(c*h*
q) + 3*d*g*(3*p + q)))*r - 6*d^2*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*
h^2)*q^2*r^2*Log[c + d*x]^2 - 6*q*r*Log[c + d*x]*((6*a^2*c*d^2*h^2*p - 3*a*
b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q) - 9*c*d
*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 - 3*c*d*g*h
+ c^2*h^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2*d^2*h^2*p*
(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p + q)*(54*g
+ 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3*a*b*p*(-12*c
^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*(p + q)*x + 5*
h^2*(p + q)*x^2))) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6*g^2 - 6*g*h*x
- h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d^2*(p + q)*(18*g
^2 + 9*g*h*x + 2*h^2*x^2)))*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^2*d
^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) +
 36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(54*b
^3*d^3)
```

fricas [F] time = 1.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(h^2x^2+2\,ghx+g^2\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{2} \ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.18, size = 1123, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b
^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
 (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
 c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*
g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*log(((b*x + a
)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3*(6*c*d
^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2*g^2 - 9
*(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*log(d*x +
 c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h*p*q + a^3
*d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2
*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*h^2*x^
3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2*p*q)*b^3*lo
g(b*x + a)*log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*f^2*g*h*q^2 + c
^3*f^2*h^2*q^2)*b^3*log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2*d^3*f^2*h^2 - (
9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^2*h^2)*b^3)*x^2 -
 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 + a^3*d^3*f^2*h^2*p^
2)*log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^2 + 3*(2*c*d^2*f^2*
h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^2 + 2*p*q + q^2)*d^3*f^
2*g^2 - 27*(p*q + q^2)*c*d^2*f^2*g*h + (8*p*q + 11*q^2)*c^2*d*f^2*h^2)*b^3)
```

```
*x - 6*((11*p^2 + 2*p*q)*a^3*d^3*f^2*h^2 + 3*(c*d^2*f^2*h^2*p*q - 3*(3*p^2
+ p*q)*d^3*f^2*g*h)*a^2*b - 6*(3*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q - 3*
(p^2 + p*q)*d^3*f^2*g^2)*a*b^2)*log(b*x + a))/(b^3*d^3))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\left(g+h\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g+hx\right)^2 \log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((g + h*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.37 $\int (g + hx) \log^2\left(e\left(f(a + bx)^p(c + dx)^q\right)^r\right) dx$

Optimal. Leaf size=1063

$$-\frac{p^2r^2\log^2(a+bx)(bg-ah)^2}{2b^2h}+\frac{pqr^2\log(a+bx)(bg-ah)^2}{2b^2h}-\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^2}{b^2h}+\frac{pr\log(a+\cdots}{\cdots}$$

```
[Out] 1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+3/2*(-a*h+b*g)*p*q*r^2*x/
b+3/2*(-c*h+d*g)*p*q*r^2*x/d-(-c*h+d*g)*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d-(-a*h
+b*g)*p*q*r^2*(d*x+c)*ln(d*x+c)/b/d-(-a*h+b*g)^2*p*q*r^2*ln(-d*(b*x+a)/(-a*
d+b*c))*ln(d*x+c)/b^2/h-(-c*h+d*g)^2*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b
*c))/d^2/h+1/2*(-a*h+b*g)^2*p*q*r^2*ln(b*x+a)/b^2/h+1/2*(-c*h+d*g)^2*p*q*r^
2*ln(d*x+c)/d^2/h+(-a*h+b*g)^2*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2/h+(-c*h+d*g)^2*q*r*ln(d*x+c)*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h-(-c*h+d*g)^2*p*q*
r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^2/h-(-a*h+b*g)^2*p*q*r^2*polylog(2,b
*(d*x+c)/(-a*d+b*c))/b^2/h+1/2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+1/2*q*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d
*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h-2*(-a*h+b*g)*p^2*r^2*(b*x+a)*ln(b*
x+a)/b^2-1/2*h*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^2-1/2*p*q*r^2*(h*x+g)^2*ln(b*x
+a)/h-1/2*(-a*h+b*g)^2*p^2*r^2*ln(b*x+a)^2/b^2/h-2*(-c*h+d*g)*q^2*r^2*(d*x+
c)*ln(d*x+c)/d^2-1/2*h*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^2-1/2*p*q*r^2*(h*x+g)^
2*ln(d*x+c)/h-1/2*(-c*h+d*g)^2*q^2*r^2*ln(d*x+c)^2/d^2/h+(-a*h+b*g)*p*r*x*(
p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b+(-c*h+d*g)*q
*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d+1/2*p*
q*r^2*(h*x+g)^2/h+1/4*p^2*r^2*(b*h*x-3*a*h+4*b*g)^2/b^2/h+1/4*q^2*r^2*(d*h*
x-3*c*h+4*d*g)^2/d^2/h
```

Rubi [A] time = 1.16, antiderivative size = 1097, normalized size of antiderivative = 1.03, number of steps used = 39, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

$$\frac{p^2r^2\log^2(a+bx)(bg-ah)^2}{2b^2h}+\frac{pqr^2\log(a+bx)(bg-ah)^2}{2b^2h}-\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^2}{b^2h}+\frac{pr\log(a+b\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
```

```
b*x])/(2*b^2*h) - ((d*g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (p*q
*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) + ((b*g - a*h)^2*p^2*r^2*Log[a + b*x]^
2)/(2*b^2*h) - (p^2*r^2*Log[a + b*x]*((4*h*(b*g - a*h)*(a + b*x))/b^2 + (h^
2*(a + b*x)^2)/b^2 + (2*(b*g - a*h)^2*Log[a + b*x])/b^2))/(2*h) + ((d*g - c
*h)^2*p*q*r^2*Log[c + d*x])/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[
c + d*x])/(b*d) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) + ((d*g -
c*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - (q^2*r^2*Log[c + d*x]*((4*h*(d*g
 - c*h)*(c + d*x))/d^2 + (h^2*(c + d*x)^2)/d^2 + (2*(d*g - c*h)^2*Log[c + d
*x])/d^2))/(2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b
*c - a*d)])/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*
x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/b + ((d*g - c*h)*q*r*x*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/d + (
p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c +
 d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h) + ((b*g - a*h)^2*p*r*L
og[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x]
+ q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(d^2*h) + ((g +
 h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h) - ((d*g - c*h)^2*p*q*
r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h) - ((b*g - a*h)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int \frac{(g+hx)^2\log\left(e\left(f(a\right.\right.}{a+}}{h}$$

$$= \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{\left(bp^2r^2\right)\int \frac{(g+hx)^2\log(a+}{a+bx}}{h}$$

$$= \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}\right.}{}\right.}{}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a+bx)}{b^2}+\frac{h^2(a+bx)^2}{b^2}+\frac{2(bg-ah)^2\log(a+bx)}{b^2}\right)}{2h} -$$

$$= -\frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a+bx)}{b^2}+\right.}{2h}$$

$$= \frac{(bg-ah)pqr^2x}{b} + \frac{(dg-ch)pqr^2x}{d} - \frac{(dg-ch)pqr^2(a+bx)\log(a+}{bd}$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg}{}$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg}{}$$

<u>**Mathematica**</u> [**A**] time = 0.93, size = 480, normalized size = 0.45

$$\frac{2pr\log(a+bx)\left(ad\left((4bdg-2adh)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+adhr(3p+q)+2bqr(ch-2dg)\right)-2qr(bc-ad\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d^2*(-2*b*g + a*h)*p^2*r^2*Log[a + b*x]^2 + 2*p*r*Log[a + b*x]*(2*b^2*
c*(-2*d*g + c*h)*q*r*Log[c + d*x] - 2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h
)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b*(-2*d*g + c*h)*q*r + a*d*h*
(3*p + q)*r + (4*b*d*g - 2*a*d*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) +
b*(2*b*c*(-2*d*g + c*h)*q^2*r^2*Log[c + d*x]^2 + 2*q*r*Log[c + d*x]*(2*a*d*
(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-2*d*g
```

```
+ c*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-2*a*p*(-4*d*g*q + 2
*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g + h*x)))
 - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x)))*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]^2)) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p*q*r^2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b^2*d^2)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(hx+g\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
[Out] integral((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 1.09, size = 623, normalized size = 0.59

$$\frac{1}{2}\left(hx^{2}+2\,gx\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}+\frac{r\left(\frac{2\left(2\,abfgp-a^{2}fhp\right)\log\left(bx+a\right)}{b^{2}}+\frac{2\left(2\,cdfgq-c^{2}fhq\right)\log\left(dx+c\right)}{d^{2}}-\frac{bdfh\left(p+q\right)x^{2}-2 \cdots}{\cdots}\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*r*(2*(2*a*
b*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x
 + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*
q)*b)*x)/(b*d))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*(2*a*c*
d*f^2*h*p*q - (4*(p*q + q^2)*c*d*f^2*g - (p*q + 3*q^2)*c^2*f^2*h)*b)*log(d*
x + c)/(b*d^2) - 4*(2*a*b*d^2*f^2*g*p*q - a^2*d^2*f^2*h*p*q - (2*c*d*f^2*g*
p*q - c^2*f^2*h*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^2*d^2) + ((p^2 + 2*p*q + q^2)*b^2*d
^2*f^2*h*x^2 - 4*(2*c*d*f^2*g*p*q - c^2*f^2*h*p*q)*b^2*log(b*x + a)*log(d*x
 + c) - 2*(2*c*d*f^2*g*q^2 - c^2*f^2*h*q^2)*b^2*log(d*x + c)^2 - 2*(2*a*b*d
^2*f^2*g*p^2 - a^2*d^2*f^2*h*p^2)*log(b*x + a)^2 - 2*(3*(p^2 + p*q)*a*b*d^2
*f^2*h - (4*(p^2 + 2*p*q + q^2)*d^2*f^2*g - 3*(p*q + q^2)*c*d*f^2*h)*b^2)*x
 + 2*((3*p^2 + p*q)*a^2*d^2*f^2*h + 2*(c*d*f^2*h*p*q - 2*(p^2 + p*q)*d^2*f^
2*g)*a*b)*log(b*x + a))/(b^2*d^2))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,\left(g+h\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g + hx\right) \log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral((g + h*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.38 $\int \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=269

$$\frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{2r(p+q)(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{2qr(bc-ad)\log(c+}{}$$

```
[Out] 2*(p+q)^2*r^2*x-2*(-a*d+b*c)*q*(p+q)*r^2*ln(d*x+c)/b/d-2*(-a*d+b*c)*p*q*r^2
*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d-(-a*d+b*c)*q^2*r^2*ln(d*x+c)^2/b/d
-2*(p+q)*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2*(-a*d+b*c)*q*r*ln(d*
x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)^2/b-2*(-a*d+b*c)*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d
```

Rubi [A] time = 0.15, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2487, 2494, 2394, 2393, 2391, 2390, 2301, 31, 8}

$$-\frac{2pqr^2(bc-ad)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}+\frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{2r(p+q)(a+bx)\log\left(e\left(f(a+b\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] 2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*Log[c + d*x])/(b*d) - (2*(
b*c - a*d)*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d) -
((b*c - a*d)*q^2*r^2*Log[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*d) + ((a + b*x)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} + \frac{(2(bc-ad)qr)\int \frac{\log\left(e\left(f(a+bx)^p(\right.\right.}{c+dx}}{b}$$

$$= -\frac{2(p+q)r(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} + \frac{2(bc-ad)qr\log(c-}{}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log\left(-\right.}{bd}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log\left(-\right.}{bd}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log\left(-\right.}{bd}$$

Mathematica [A] time = 0.23, size = 389, normalized size = 1.45

$$\frac{bdx\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2pr\log(a+bx)\left(ad\left(qr-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right) + qr(ad-bc)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d*p*q*r^2 + 2*b*d*p^2*r^2*x + 4*b*d*p*q*r^2*x + 2*b*d*q^2*r^2*x - a*d*
p^2*r^2*Log[a + b*x]^2 - 2*b*c*p*q*r^2*Log[c + d*x] + 2*a*d*p*q*r^2*Log[c +
 d*x] - 2*b*c*q^2*r^2*Log[c + d*x] - b*c*q^2*r^2*Log[c + d*x]^2 - 2*p*r*Log
[a + b*x]*(b*c*q*r*Log[c + d*x] + (-(b*c) + a*d)*q*r*Log[(b*(c + d*x))/(b*c
 - a*d)] + a*d*(q*r - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 2*a*d*p*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*p*r*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] - 2*b*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*c*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + b*d*x*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)])/(b*d)
```

fricas [F] time = 1.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

maxima [A] time = 0.88, size = 298, normalized size = 1.11

$$x\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 - \frac{2\left(f(p+q)x - \frac{afp\log(bx+a)}{b} - \frac{cfq\log(dx+c)}{d}\right)r\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{f} - \frac{\left(\frac{2(pq}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*log(b*x +
 a)/b - c*f*q*log(d*x + c)/d)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - (2
*(p*q + q^2)*c*f^2*log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
```

```
)/(b*d) + (a*d*f^2*p^2*log(b*x + a)^2 + 2*b*c*f^2*p*q*log(b*x + a)*log(d*x
+ c) + b*c*f^2*q^2*log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x + 2*(p^
2 + p*q)*a*d*f^2*log(b*x + a))/(b*d))*r^2/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(e\left(f\,(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

3.39 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

Optimal. Leaf size=1471

$$\frac{pq\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)r^2}{h}+\frac{p^2\log^2(a+bx)\log(g+hx)r^2}{h}+\frac{q^2\log^2(c+dx)\log(g+hx)r^2}{h}+\frac{2pq\log(a}{}$$

```
[Out] -2*p*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-2*q*r*ln(d*x+c
)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h+2*p*r*ln(b*x+a)*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*ln(b*(h*x+g)/(-a*h+b*g))/h+2*q*r*ln(d*x+c)*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln((-a*h+b*g)*(d*x+
c)/(-c*h+d*g)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/h-2*p*q*r^2*ln((-a*h+
b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))*polylog(2,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b
*x+a))/h+p*q*r^2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(
b*x+a))^2/h+p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(b*(h*x+g)/(-a*h+b*g))/h+
p*q*r^2*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(b*(h*x+g)/(-a*h+b*g)
)/h-p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(d*(h*x+g)/(-c*h+d*g))/h-p*q*r^2*
ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(-(-a*d+b*c)*(h*x+g)/(-c*h+d*
g)/(b*x+a))/h-2*q^2*r^2*polylog(3,-h*(d*x+c)/(-c*h+d*g))/h+ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)^2*ln(h*x+g)/h+2*p*q*r^2*ln(b*x+a)*ln(d*x+c)*ln(h*x+g)/h-2*p
*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*g))*ln(b*(h*x+g)/(-a*h+b*g))/h-2*p*q
*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))*ln
(b*(h*x+g)/(-a*h+b*g))/h-2*p*q*r^2*ln(b*x+a)*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d
*g))/h+2*p*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*g))*ln(d*(h*x+g)/(-c*h+d*g
))/h+2*p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(
b*x+a))*ln(d*(h*x+g)/(-c*h+d*g))/h+p^2*r^2*ln(b*x+a)^2*ln(h*x+g)/h+q^2*r^2*
ln(d*x+c)^2*ln(h*x+g)/h-p^2*r^2*ln(b*x+a)^2*ln(b*(h*x+g)/(-a*h+b*g))/h-q^2*
r^2*ln(d*x+c)^2*ln(d*(h*x+g)/(-c*h+d*g))/h-2*p*r*(q*r*ln((-a*h+b*g)*(d*x+c)
/(-c*h+d*g)/(b*x+a))-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*polylog(2,-h*(b*x+a)/
(-a*h+b*g))/h+2*q*r*(p*r*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))+ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r))*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h-2*p^2*r^2*polyl
og(3,-h*(b*x+a)/(-a*h+b*g))/h-2*p*q*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-
2*p*q*r^2*polylog(3,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,b*(d*x+c)/
d/(b*x+a))/h+2*p*q*r^2*polylog(3,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))/h
```

Rubi [A] time = 1.93, antiderivative size = 2096, normalized size of antiderivative = 1.42, number of steps used = 29, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2497, 2500, 2394, 2393, 2391, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]

[Out] -((Log[(a + b*x)^(p*r)]^2*Log[g + h*x])/h) - (2*p*q*r^2*Log[-((d*(a + b*x))
/(b*c - a*d))]*Log[c + d*x]*Log[g + h*x])/h - (2*p*q*r^2*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)]*Log[g + h*x])/h + (2*q*r*(p*r*Log[a + b*x] - Log[
(a + b*x)^(p*r)])*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (2*p*
r*Log[-((h*(a + b*x))/(b*g - a*h))]*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)
])*Log[g + h*x])/h - (Log[(c + d*x)^(q*r)]^2*Log[g + h*x])/h + (2*p*r*Log[-
((h*(a + b*x))/(b*g - a*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (2*q*r*Log[-((h*(c
 + d*x))/(d*g - c*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]^2*Log[g + h*x])/h + (Log[(a + b*x)^(p*r)]^2*Log[(b*(g + h*x))
/(b*g - a*h)])/h + (Log[(c + d*x)^(q*r)]^2*Log[(d*(g + h*x))/(d*g - c*h)])/
h - (p*q*r^2*(Log[(b*(c + d*x))/(b*c - a*d)] + Log[(b*g - a*h)/(b*(g + h*x)
)] - Log[((b*g - a*h)*(c + d*x))/((b*c - a*d)*(g + h*x))])*Log[-(((b*c - a*
d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]^2)/h + (p*q*r^2*(Log[(b*(c + d*x))/
(b*c - a*d)] - Log[-((h*(c + d*x))/(d*g - c*h))])*(Log[a + b*x] + Log[-(((b
*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])^2)/h - (p*q*r^2*(Log[-((d*(
a + b*x))/(b*c - a*d))] + Log[(d*g - c*h)/(d*(g + h*x))] - Log[-(((d*g - c*
h)*(a + b*x))/((b*c - a*d)*(g + h*x)))])*Log[((b*c - a*d)*(g + h*x))/((b*g
- a*h)*(c + d*x))]^2)/h + (p*q*r^2*(Log[-((d*(a + b*x))/(b*c - a*d))] - Log
[-((h*(a + b*x))/(b*g - a*h))])*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x))
/((b*g - a*h)*(c + d*x))])^2)/h - (2*p*q*r^2*(Log[g + h*x] - Log[-(((b*c -
a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])*PolyLog[2, -((d*(a + b*x))/(b*c
- a*d))])/h + (2*p*r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((h*(a + b*x))/(b*g -
 a*h))])/h - (2*p*q*r^2*(Log[g + h*x] - Log[((b*c - a*d)*(g + h*x))/((b*g -
 a*h)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/h + (2*q*r*Log[(c
 + d*x)^(q*r)]*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*Log
[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]*PolyLog[2, (h*(a + b*x
))/(b*(g + h*x))])/h - (2*p*q*r^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h
)*(a + b*x)))]*PolyLog[2, -(((d*g - c*h)*(a + b*x))/((b*c - a*d)*(g + h*x))
)])/h + (2*p*q*r^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]*Pol
yLog[2, (h*(c + d*x))/(d*(g + h*x))])/h - (2*p*q*r^2*Log[((b*c - a*d)*(g +
h*x))/((b*g - a*h)*(c + d*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((b*c - a
*d)*(g + h*x))])/h + (2*p*r*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)])*PolyL
og[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*p*r*(Log[(a + b*x)^(p*r)] + Log[(c
 + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(g + h
*x))/(b*g - a*h)])/h - (2*p*q*r^2*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x
))/((b*g - a*h)*(c + d*x))])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*
q*r*(p*r*Log[a + b*x] - Log[(a + b*x)^(p*r)])*PolyLog[2, (d*(g + h*x))/(d*g
 - c*h)])/h + (2*q*r*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h - (
2*p*q*r^2*(Log[a + b*x] + Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b
*x)))])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h + (2*p*q*r^2*PolyLog[3, -(
```

```
(d*(a + b*x))/(b*c - a*d))])/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/(b*g
 - a*h))])/h + (2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/h - (2*q^2
*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*PolyLog[3, (h
*(a + b*x))/(b*(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, -(((d*g - c*h)*(a + b
*x))/((b*c - a*d)*(g + h*x)))])/h + (2*p*q*r^2*PolyLog[3, (h*(c + d*x))/(d*
(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((b*c - a*d)
*(g + h*x))])/h + (2*p*q*r^2*PolyLog[3, (b*(g + h*x))/(b*g - a*h)])/h + (2*
p*q*r^2*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
```

```
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2497

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2)/h, x] + (-Dist[(2*b*p*r)/h, Int[(Log[g + h*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(a + b*x), x], x] - Dist[(2*d*q*r)/h,
 Int[(Log[g + h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(c + d*x), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx\right.\right.}{a+bx}}{h}$$

$$= \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log((a+bx)^{pr})\log(g+h}{a+bx}}{h}$$

$$= \frac{2pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e\left(f(a+bx\right.\right.\right.}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{\log^2\left((c+dx)^{qr}\right)\log(g+hx)}{h} + \frac{2pr\log}{}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g}{h}$$

Mathematica [A] time = 0.29, size = 1370, normalized size = 0.93

$$\frac{pq\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)r^2+p^2\log^2(a+bx)\log(g+hx)r^2+q^2\log^2(c+dx)\log(g+hx)r^2+2pq\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

```
[Out] (p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2 + p^2*r^2*Log[a + b*x]^2*Log[g + h*x] + 2*p*q*r^2*Lo
g[a + b*x]*Log[c + d*x]*Log[g + h*x] + q^2*r^2*Log[c + d*x]^2*Log[g + h*x]
- 2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] - 2*
q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x] - p^2*r^2*Log[a + b*x]^2*Log[
```

```
(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*
g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[(h*(c + d*x))/(-(d*
g) + c*h)]^2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*
(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2*Log[(b*(g + h*x))/(b*g - a*h)] + 2*p*r*Log[a + b*x]*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[
a + b*x]*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] - q^2*r^2*Log[c + d*x]
^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[(h*(c +
d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
Log[(d*(g + h*x))/(d*g - c*h)] + 2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x))] + 2*p*r*(-(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(a + b*x))/
(-(b*g) + a*h)] + 2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a +
b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(c + d*x))/(-(
d*g) + c*h)] + 2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - 2*p^2*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*
p*q*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*p*q*r^2*PolyLog[3, (h*
(c + d*x))/(-(d*g) + c*h)] - 2*q^2*r^2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] + 2*p*q*r^2*PolyLo
g[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])/h
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)^{2}}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="maxima")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)^{2}}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^{p}\,(c+dx)^{q}\right)^{r}\right)^{2}}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g),x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(g + h*x), x)
```

3.40 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=832

$$\frac{2bpq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)r^2}{h(bg-ah)}+\frac{2dpq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)r^2}{h(dg-ch)}-\frac{2dpq\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)r^2}{h(dg-ch)}-\frac{2bpq\log\cdots}{\cdots}$$

```
[Out] 2*b*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)+2*d*p*q*r^2*ln
(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)-2*b*p*r*ln(b*x+a)*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)-2*d*q*r*ln(
d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*
h+d*g)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)+2*b*p*r*(p*r*ln(b*x+a)+q
*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)+2*d*q*
r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h
/(-c*h+d*g)-2*d*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)-2*b
*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)-2*b*p^2*r^2*ln(b*x
+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)-2*d*q^2*r^2*ln(d*x+c)*ln(1+(-c*
h+d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h
/(-a*h+b*g)+2*d*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)-2*d*p
*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)+2*d*q^2*r^2*polylog(2,
(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c
))/h/(-a*h+b*g)-2*b*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)
```

Rubi [A] time = 0.93, antiderivative size = 878, normalized size of antiderivative = 1.06, number of steps used = 35, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2498, 2513, 2411, 2344, 2301, 2317, 2391, 2418, 2394, 2393, 36, 31}

$$\frac{bp^2\log^2(a+bx)r^2}{h(bg-ah)}+\frac{dq^2\log^2(c+dx)r^2}{h(dg-ch)}+\frac{2bpq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)r^2}{h(bg-ah)}+\frac{2dpq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)r^2}{h(dg-ch)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (b*p^2*r^2*Log[a + b*x]^2)/(h*(b*g - a*h)) + (2*b*p*q*r^2*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)) + (d*q^2*r^2*Log[c + d*x]^2)
/(h*(d*g - c*h)) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]
)/(h*(d*g - c*h)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)) - (2*d*q*r*Log
[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(h*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g
```

```
 + h*x)) + (2*b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a +
 b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*
x])/(h*(d*g - c*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*
h)])/(h*(b*g - a*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a
*h)])/(h*(d*g - c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g -
c*h)])/(h*(b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g -
 c*h)])/(h*(d*g - c*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*
d))])/(h*(d*g - c*h)) - (2*b*p^2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h)
)])/(h*(b*g - a*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))]
)/(h*(d*g - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*
(b*g - a*h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b
*g - a*h)) - (2*d*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(d*g
 - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)}\,dx}{h} + \frac{(2$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{\left(2bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)}\,dx}{h} + \frac{\left(2bpqr^2\right)}{}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{\left(2p^2r^2\right)\mathrm{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}\,dx, x, a+\right.}{h}$$

$$= -\frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c\right.\right.\right.}{h(bg-ah)}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+}{h(dg-ch)}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+}{h(dg-ch)}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+}{h(dg-ch)}$$

Mathematica [B] time = 1.34, size = 2930, normalized size = 3.52

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]

[Out] (-(b*d*g^2*p^2*r^2*Log[a + b*x]^2) + b*c*g*h*p^2*r^2*Log[a + b*x]^2 - b*d*g
*h*p^2*r^2*x*Log[a + b*x]^2 + b*c*h^2*p^2*r^2*x*Log[a + b*x]^2 - 2*b*d*g^2*
p*q*r^2*Log[a + b*x]*Log[c + d*x] + 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[c +
d*x] - 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[c + d*x] + 2*a*d*h^2*p*q*r^2*x*
Log[a + b*x]*Log[c + d*x] - b*d*g^2*q^2*r^2*Log[c + d*x]^2 + a*d*g*h*q^2*r^
2*Log[c + d*x]^2 - b*d*g*h*q^2*r^2*x*Log[c + d*x]^2 + a*d*h^2*q^2*r^2*x*Log
[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*
h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + 2*
b*c*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - 2*a*d*h^
2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - b*c*g*h*p*q*r^
2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + a*d*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]^2 - b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 +
a*d*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + 2*b*c*g*h*p*q*r^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))] - 2*a*d*g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g
- a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(-(b*c
) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))
] - 2*a*d*h^2*p*q*r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d
*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/
((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*
r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - b*c*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))]^2 + a*d*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 - b*c*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + a*d*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + 2*b*d*g^2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - 2*b*c*g*h*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*
b*d*g*h*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*c*h^2
*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g^2*q*r*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*g*h*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g*h*q*r*x*Log[c + d*x]*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*h^2*q*r*x*Log[c + d*x]*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] - b*d*g^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 +
 b*c*g*h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*d*g*h*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r]^2 - a*c*h^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 2*
b*d*g^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*g*h*p*q
*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*q*r^2*x*Log[
a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*
Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g
) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*
```

```
x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g*h*p*q*r^2*x*Lo
g[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*h^2*
p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)]
- 2*b*d*g^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g
 - a*h)] + 2*b*c*g*h*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h
*x))/(b*g - a*h)] - 2*b*d*g*h*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Lo
g[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*h^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[a + b*x]*Lo
g[(d*(g + h*x))/(d*g - c*h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g
 - c*h)] - 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)]
- 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*
g - c*h)] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g +
 h*x))/(d*g - c*h)] - 2*b*d*g*h*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]
*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(
d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g^2*q*r*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*d*g*h*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g*h*q
*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] +
2*a*d*h^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g
 - c*h)] + 2*p*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^2*(g + h*x)*PolyLog[2,
 (h*(a + b*x))/(-(b*g) + a*h)] + 2*q*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^
2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] + 2*b*c*g*h*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*a*d*g*h*p*q*r^2*PolyLog[2, (b*(c
+ d*x))/(d*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))] - 2*a*d*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(h
*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{h^2x^2+2\,ghx+g^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{\left(hx+g\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{\left(hx+g\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

maxima [A] time = 1.80, size = 745, normalized size = 0.90

$$\frac{2\left(\frac{bfp\log(bx+a)}{bg-ah}+\frac{dfq\log(dx+c)}{dg-ch}-\frac{\left(adfhq-\left(dfg\left(p+q\right)-cfhp\right)b\right)\log\left(hx+g\right)}{\left(dgh-ch^2\right)a-\left(dg^2-cgh\right)b}\right)r\log\left(\left(\left(bx+a\right)^p(dx+c)^q f\right)^r e\right)}{fh}-\frac{\left(\frac{2\left(bcf^2hpq-adf^2hpq\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] 2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*f
*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^
2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^2*h
*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) +
2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x + a)*log
((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*
h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q
+ q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog
```

```
(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) - ((d
*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q - a*d*f^2*h*p
*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)*log(d*x + c
)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*log(b*x +
a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*log(d*x + c))*
log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(f^2*h) - log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=1304

$$\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^2}{h(bg-ah)^2}-\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.\right.}{h(bg-ah)^2}$$

```
[Out] b^2*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^2+b
^2*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h/(-a*h+b*g)^2+d^2*q^2*r^2*polylo
g(2,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)^2-1/2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)
^2/h/(h*x+g)^2+d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)/(h*x+g)+b*p*q*r^2*ln(d*x+c)
/h/(-a*h+b*g)/(h*x+g)-b*d*p*q*r^2*ln(b*x+a)/h/(-a*h+b*g)/(-c*h+d*g)-b*d*p*q
*r^2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h+d*g)+2*b*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)
/(-c*h+d*g)-b*p^2*r^2*(b*x+a)*ln(b*x+a)/(-a*h+b*g)^2/(h*x+g)-d*q^2*r^2*(d*x
+c)*ln(d*x+c)/(-c*h+d*g)^2/(h*x+g)-b*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)/(h*x+g)-d*q*r*(p*r*ln(b*x+a)+q*r*l
n(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)/(h*x+g)-b^2*p*r*ln(b
*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h
+b*g)^2-d^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r))/h/(-c*h+d*g)^2+b^2*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q*r*(p*r*ln(b*x+a)+q*r*
ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^2-b^2*p^2
*r^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)^2-d^2*q^2*r^2*ln(d*x
+c)*ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^2+d^2*p*q*r^2*polylog(2,-d*(b*x
+a)/(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))
/h/(-c*h+d*g)^2+b^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^2-
b^2*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)^2+b^2*p*q*r^2*ln(
-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^2+d^2*p*q*r^2*ln(b*x+a)*ln(b*
(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h
+b*g))/h/(-c*h+d*g)^2-b^2*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*
h+b*g)^2
```

Rubi [A] time = 1.41, antiderivative size = 1362, normalized size of antiderivative = 1.04, number of steps used = 47, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2418, 2394, 2393, 2395, 36, 44}

$$\frac{p^2r^2\log^2(a+bx)b^2}{2h(bg-ah)^2}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^2}{h(bg-ah)^2}-\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\right.\right.}{h(bg-ah)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[
a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((
b*g - a*h)^2*(g + h*x)) + (b^2*p^2*r^2*Log[a + b*x]^2)/(2*h*(b*g - a*h)^2)
- (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g - c*h)) + (b*p*q*r^2*Log[c
 + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c + d*x)*Log[c + d*x])/((d
*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log
[c + d*x])/(h*(b*g - a*h)^2) + (d^2*q^2*r^2*Log[c + d*x]^2)/(2*h*(d*g - c*h
)^2) + (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g -
c*h)^2) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]))/(h*(b*g - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] +
q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)*(g
 + h*x)) - (b^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*
x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r]))/(h*(d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*h*(g +
h*x)^2) + (b^2*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log
[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g
 - c*h)^2) + (b^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g
+ h*x])/(h*(d*g - c*h)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*
g - a*h)])/(h*(b*g - a*h)^2) - (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/
(b*g - a*h)])/(h*(d*g - c*h)^2) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x
))/(d*g - c*h)])/(h*(b*g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[(d*(g +
h*x))/(d*g - c*h)])/(h*(d*g - c*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b
*x))/(b*c - a*d))])/(h*(d*g - c*h)^2) - (b^2*p^2*r^2*PolyLog[2, -((h*(a + b
*x))/(b*g - a*h))])/(h*(b*g - a*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b
*x))/(b*g - a*h))])/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(h*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x)
)/(d*g - c*h))])/(h*(b*g - a*h)^2) - (d^2*q^2*r^2*PolyLog[2, -((h*(c + d*x)
)/(d*g - c*h))])/(h*(d*g - c*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(d$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{\left(bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{\left(bpqr^2\right)}{}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}\,dx, x, a\right.}{h}$$

$$= -\frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{h(bg-ah)(g+hx)}$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} -$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2\log^2(a+bx)}{2h(bg-ah)^2} -$$

$$= -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)}$$

Mathematica [B] time = 6.31, size = 15960, normalized size = 12.24

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{h^3x^3+3\,gh^2x^2+3\,g^2hx+g^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^3*x^3 + 3*g*h^2*x^2 + 3*
g^2*h*x + g^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^3, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

maxima [A] time = 3.86, size = 1857, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="maxima
")

[Out] (b^2*f*p*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*log(d*x + c
)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q -
(d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*log(h*x + g)/((d^2*g
^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*
h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (d*f*g
*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g
*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*log(((b*x + a)^p*(d*x + c)^q
*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q - (2*c
*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h^3)*a^2 -
2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^2*f^2*g*h*p
*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2 - (p^2 + p*
q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dil
og(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h
^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2 - a^2*d^2*f^2*h
^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g^2)*b^
2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/
(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2
*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2)
- 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*d^2*f^2*g)*b)*log(d*x +
 c)/((d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*a - (d^2*g^3 - 2*c*d*g^2*h + c^2*g
*h^2)*b) + 2*(a^2*d^2*f^2*h^2*q^2 + 2*(c*d*f^2*h^2*p*q - (p*q + q^2)*d^2*f^
2*g*h)*a*b + (c^2*f^2*h^2*p^2 + (p^2 + 2*p*q + q^2)*d^2*f^2*g^2 - 2*(p^2 +
p*q)*c*d*f^2*g*h)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + ((d^3*f^2*g^3*p^2 - 3*c*d^2*f^2*g^2*h*p^2 + 3*c^2*d
*f^2*g*h^2*p^2 - c^3*f^2*h^3*p^2)*b^2*log(b*x + a)^2 - 2*(b^2*d^2*f^2*g^2*p
*q - 2*a*b*d^2*f^2*g*h*p*q + a^2*d^2*f^2*h^2*p*q)*log(b*x + a)*log(d*x + c)
 - (b^2*d^2*f^2*g^2*q^2 - 2*a*b*d^2*f^2*g*h*q^2 + a^2*d^2*f^2*h^2*q^2)*log(
d*x + c)^2 + 2*((d^2*f^2*g*h*p*q - c*d*f^2*h^2*p*q)*a*b - (c^2*f^2*h^2*p^2
+ (p^2 + p*q)*d^2*f^2*g^2 - (2*p^2 + p*q)*c*d*f^2*g*h)*b^2)*log(b*x + a) -
2*((2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*
f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*log(b*x + a) + (2*a*b*d^2*f^2*g
*h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q
+ q^2)*d^2*f^2*g^2)*b^2)*log(d*x + c))*log(h*x + g))/((d^2*g^2*h^2 - 2*c*d*
```

```
g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2
*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2))*r^2/(f^2*h) - 1/2*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)^2/((h*x + g)^2*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{\left(g+h\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=1957

$$-\frac{p^2r^2\log(a+bx)b^3}{3h(bg-ah)^3}+\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^3}{3h(bg-ah)^3}-\frac{2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\right.}{3h(bg-ah)^3}$$

```
[Out] -1/3*b^3*p^2*r^2*ln(b*x+a)/h/(-a*h+b*g)^3-1/3*d^3*q^2*r^2*ln(d*x+c)/h/(-c*h
+d*g)^3+b^3*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^3+d^3*q^2*r^2*ln(h*x+g)/h/(-c*h+
d*g)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)^3+2/3*d^3*p*q*r^2*po
lylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)^3-2/3*d^3*p*q*r^2*polylog(2,-h*
(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)^3+2/3*b^3*p*q*r^2*polylog(2,b*(d*x+c)/(-a*
d+b*c))/h/(-a*h+b*g)^3-2/3*b^3*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(
-a*h+b*g)^3-2/3*b*d*p*q*r^2/h/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)-1/3*b^2*p^2*r^2
/h/(-a*h+b*g)^2/(h*x+g)-1/3*d^2*q^2*r^2/h/(-c*h+d*g)^2/(h*x+g)+2/3*b^3*p^2*
r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h/(-a*h+b*g)^3+2/3*d^3*q^2*r^2*polylog(2
,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)^3-2/3*b*d^2*p*q*r^2*ln(b*x+a)/h/(-a*h+b*
g)/(-c*h+d*g)^2-1/3*b^2*d*p*q*r^2*ln(b*x+a)/h/(-a*h+b*g)^2/(-c*h+d*g)-1/3*b
*d^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h+d*g)^2-2/3*b^2*d*p*q*r^2*ln(d*x+c
)/h/(-a*h+b*g)^2/(-c*h+d*g)+b*d^2*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*g)
^2+b^2*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)^2/(-c*h+d*g)+1/3*d*q^2*r^2*ln(d*x+c
)/h/(-c*h+d*g)/(h*x+g)^2-2/3*d^2*q^2*r^2*(d*x+c)*ln(d*x+c)/(-c*h+d*g)^3/(h*
x+g)-1/3*b*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)
)/h/(-a*h+b*g)/(h*x+g)^2-1/3*d*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)/(h*x+g)^2-2/3*b^2*p*r*(p*r*ln(b*x+a)+q*r
*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)^2/(h*x+g)-2/3*d^2*
q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d
*g)^2/(h*x+g)-2/3*b^3*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)^3-2/3*d^3*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+
q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^3+2/3*b^3*p*r*(
p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-
a*h+b*g)^3+2/3*d^3*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+
c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^3-2/3*b^3*p^2*r^2*ln(b*x+a)*ln(1+(-a*h+b*g
)/h/(b*x+a))/h/(-a*h+b*g)^3-2/3*d^3*q^2*r^2*ln(d*x+c)*ln(1+(-c*h+d*g)/h/(d*
x+c))/h/(-c*h+d*g)^3+1/3*b*p^2*r^2*ln(b*x+a)/h/(-a*h+b*g)/(h*x+g)^2-2/3*b^2
*p^2*r^2*(b*x+a)*ln(b*x+a)/(-a*h+b*g)^3/(h*x+g)-2/3*d^3*p*q*r^2*ln(b*x+a)*l
n(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)^3-2/3*b^3*p*q*r^2*ln(d*x+c)*ln(d*(h*x+
g)/(-c*h+d*g))/h/(-a*h+b*g)^3+1/3*d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)/(h*x+g)^
2+2/3*d^2*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)^2/(h*x+g)+1/3*b*p*q*r^2*ln(d*x+c)/
h/(-a*h+b*g)/(h*x+g)^2+2/3*b^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)^2/(h*x+g)+2/3
*b^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^3+2/3*d^3*p*q
*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^3
```

Rubi [A] time = 2.10, antiderivative size = 2013, normalized size of antiderivative = 1.03, number of steps used = 61, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44, 2418, 2394, 2393, 2395, 36}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] -(b^2*p^2*r^2)/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g -
a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x)) -
 (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a +
b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h*(
b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g +
h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2*p
*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a + b*
x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) + (b^3*p^2*r^2*Log[a + b*x]^2)
/(3*h*(b*g - a*h)^3) - (b*d^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(d*g -
 c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) -
 (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) + (b*p*q*r^2*Log[c + d*x])/
(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)*(
g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(g + h*x)) -
(2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g - c*h)^3*(g + h*x)) + (2*b^3
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*h*(b*g - a*h)^3
) + (d^3*q^2*r^2*Log[c + d*x]^2)/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^2*Log[a
 + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*L
og[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*
h*(b*g - a*h)*(g + h*x)^2) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b
^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r]))/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*
r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)^2*
(g + h*x)) - (2*b^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Lo
g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/
(3*h*(g + h*x)^3) + (b^3*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p
*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g +
 h*x])/(h*(b*g - a*h)^2*(d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g -
 c*h)^3) + (2*b^3*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Lo
g[g + h*x])/(3*h*(d*g - c*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[(b*(g + h
```

```
*x))/(b*g - a*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b
*(g + h*x))/(b*g - a*h)])/(3*h*(d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]
*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c
 + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^
2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*h*(d*g - c*h)^3) - (2*b^3*p^
2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(b*g - a*h)^3) - (2*d^
3*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(d*g - c*h)^3) + (
2*b^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) -
(2*b^3*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(b*g - a*h)^3
) - (2*d^3*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(d*g - c*
h)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
```

```
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{(2$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{\left(2bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{\left(2bpqr^2\right)}{}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{\left(2p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}\,dx, x, a\right.}{3h}$$

$$= -\frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{3h(bg-ah)(g+hx)^2} -$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)} +$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)} -$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g}$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g}$$

Mathematica [B] time = 6.53, size = 47127, normalized size = 24.08

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{h^4x^4+4gh^3x^3+6g^2h^2x^2+4g^3hx+g^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^4*x^4 + 4*g*h^3*x^3 + 6*
g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{(hx+g)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^4, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

maxima [B] time = 8.91, size = 4732, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*f*p*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*
h^3) + 2*d^3*f*q*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c^
3*h^3) - 2*(3*a*b^2*d^3*f*g^2*h*q - 3*a^2*b*d^3*f*g*h^2*q + a^3*d^3*f*h^3*q
 - (d^3*f*g^3*(p + q) - 3*c*d^2*f*g^2*h*p + 3*c^2*d*f*g*h^2*p - c^3*f*h^3*p
)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h
^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a
^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^
2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) + ((3*d^
2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*g*h^2*(p
+ q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p + q) - 3*
c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q - (d^2*f*g^2*
h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^
3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b
+ (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 +
 c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*
h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h
^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (
d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*r*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g
*h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3
*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^
2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3
 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^
3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g*h^2*p*q + a^
3*d^3*f^2*h^3*p*q + (3*c*d^2*f^2*g^2*h*p^2 - 3*c^2*d*f^2*g*h^2*p^2 + c^3*f^
2*h^3*p^2 - (p^2 + p*q)*d^3*f^2*g^3)*b^3)*(log(b*x + a)*log((b*h*x + a*h)/(
b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d^3*g^3*h^3 - 3*c*d^
2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3
 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*
c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4
*h^2 - c^3*g^3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*q^2 - 3*a^2*b*d^3*f^2*g
*h^2*q^2 + a^3*d^3*f^2*h^3*q^2 + (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q - (p*q + q^2)*d^3*f^2*g^3)*b^3)*(log(d*x + c)*log((d
*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^3*g^3
*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*
c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2
*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h
+ 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - (3*a^2*d^3*f^2*h^2*q^2 + (c*d^2*f^2
*h^2*p*q - (p*q + 6*q^2)*d^3*f^2*g*h)*a*b - (5*c*d^2*f^2*g*h*p*q - 2*c^2*d*
f^2*h^2*p*q - 3*(p*q + q^2)*d^3*f^2*g^2)*b^2)*log(d*x + c)/((d^3*g^3*h^2 -
3*c*d^2*g^2*h^3 + 3*c^2*d*g*h^4 - c^3*h^5)*a^2 - 2*(d^3*g^4*h - 3*c*d^2*g^3
*h^2 + 3*c^2*d*g^2*h^3 - c^3*g*h^4)*a*b + (d^3*g^5 - 3*c*d^2*g^4*h + 3*c^2*
d*g^3*h^2 - c^3*g^2*h^3)*b^2) + 3*(a^3*d^3*f^2*h^3*q^2 + (c*d^2*f^2*h^3*p*q
```

```
 - (p*q + 3*q^2)*d^3*f^2*g*h^2)*a^2*b - (4*c*d^2*f^2*g*h^2*p*q - c^2*d*f^2*
h^3*p*q - 3*(p*q + q^2)*d^3*f^2*g^2*h)*a*b^2 + (c^3*f^2*h^3*p^2 - (p^2 + 2*
p*q + q^2)*d^3*f^2*g^3 + 3*(p^2 + p*q)*c*d^2*f^2*g^2*h - (3*p^2 + p*q)*c^2*
d*f^2*g*h^2)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*
h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c
^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^
2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^
3) - ((d^3*f^2*g*h^3*q^2 - c*d^2*f^2*h^4*q^2)*a^3 - (2*c^2*d*f^2*h^4*p*q +
(2*p*q + 3*q^2)*d^3*f^2*g^2*h^2 - (4*p*q + 3*q^2)*c*d^2*f^2*g*h^3)*a^2*b -
(c^3*f^2*h^4*p^2 - (p^2 + 4*p*q + 3*q^2)*d^3*f^2*g^3*h + (3*p^2 + 8*p*q + 3
*q^2)*c*d^2*f^2*g^2*h^2 - (3*p^2 + 4*p*q)*c^2*d*f^2*g*h^3)*a*b^2 + (c^3*f^2
*g*h^3*p^2 - (p^2 + 2*p*q + q^2)*d^3*f^2*g^4 + (3*p^2 + 4*p*q + q^2)*c*d^2*
f^2*g^3*h - (3*p^2 + 2*p*q)*c^2*d*f^2*g^2*h^2)*b^3 - ((d^3*f^2*g^3*h*p^2 -
3*c*d^2*f^2*g^2*h^2*p^2 + 3*c^2*d*f^2*g*h^3*p^2 - c^3*f^2*h^4*p^2)*b^3*x +
(d^3*f^2*g^4*p^2 - 3*c*d^2*f^2*g^3*h*p^2 + 3*c^2*d*f^2*g^2*h^2*p^2 - c^3*f^
2*g*h^3*p^2)*b^3)*log(b*x + a)^2 - 2*(b^3*d^3*f^2*g^4*p*q - 3*a*b^2*d^3*f^2
*g^3*h*p*q + 3*a^2*b*d^3*f^2*g^2*h^2*p*q - a^3*d^3*f^2*g*h^3*p*q + (b^3*d^3
*f^2*g^3*h*p*q - 3*a*b^2*d^3*f^2*g^2*h^2*p*q + 3*a^2*b*d^3*f^2*g*h^3*p*q -
a^3*d^3*f^2*h^4*p*q)*x)*log(b*x + a)*log(d*x + c) - (b^3*d^3*f^2*g^4*q^2 -
3*a*b^2*d^3*f^2*g^3*h*q^2 + 3*a^2*b*d^3*f^2*g^2*h^2*q^2 - a^3*d^3*f^2*g*h^3
*q^2 + (b^3*d^3*f^2*g^3*h*q^2 - 3*a*b^2*d^3*f^2*g^2*h^2*q^2 + 3*a^2*b*d^3*f
^2*g*h^3*q^2 - a^3*d^3*f^2*h^4*q^2)*x)*log(d*x + c)^2 - (2*(d^3*f^2*g^2*h^2
*p*q - c*d^2*f^2*g*h^3*p*q)*a^2*b - (5*d^3*f^2*g^3*h*p*q - 6*c*d^2*f^2*g^2*
h^2*p*q + c^2*d*f^2*g*h^3*p*q)*a*b^2 - (3*c^3*f^2*g*h^3*p^2 - 3*(p^2 + p*q)
*d^3*f^2*g^4 + (9*p^2 + 4*p*q)*c*d^2*f^2*g^3*h - (9*p^2 + p*q)*c^2*d*f^2*g^
2*h^2)*b^3 + (2*(d^3*f^2*g*h^3*p*q - c*d^2*f^2*h^4*p*q)*a^2*b - (5*d^3*f^2*
g^2*h^2*p*q - 6*c*d^2*f^2*g*h^3*p*q + c^2*d*f^2*h^4*p*q)*a*b^2 - (3*c^3*f^2
*h^4*p^2 - 3*(p^2 + p*q)*d^3*f^2*g^3*h + (9*p^2 + 4*p*q)*c*d^2*f^2*g^2*h^2
- (9*p^2 + p*q)*c^2*d*f^2*g*h^3)*b^3)*x)*log(b*x + a) - 2*((3*a*b^2*d^3*f^2
*g^3*h*p*q - 3*a^2*b*d^3*f^2*g^2*h^2*p*q + a^3*d^3*f^2*g*h^3*p*q + (3*c*d^2
*f^2*g^3*h*p^2 - 3*c^2*d*f^2*g^2*h^2*p^2 + c^3*f^2*g*h^3*p^2 - (p^2 + p*q)*
d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*p*q - 3*a^2*b*d^3*f^2*g*h^3*p*q
 + a^3*d^3*f^2*h^4*p*q + (3*c*d^2*f^2*g^2*h^2*p^2 - 3*c^2*d*f^2*g*h^3*p^2 +
 c^3*f^2*h^4*p^2 - (p^2 + p*q)*d^3*f^2*g^3*h)*b^3)*x)*log(b*x + a) + (3*a*b
^2*d^3*f^2*g^3*h*q^2 - 3*a^2*b*d^3*f^2*g^2*h^2*q^2 + a^3*d^3*f^2*g*h^3*q^2
+ (3*c*d^2*f^2*g^3*h*p*q - 3*c^2*d*f^2*g^2*h^2*p*q + c^3*f^2*g*h^3*p*q - (p
*q + q^2)*d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*q^2 - 3*a^2*b*d^3*f^2
*g*h^3*q^2 + a^3*d^3*f^2*h^4*q^2 + (3*c*d^2*f^2*g^2*h^2*p*q - 3*c^2*d*f^2*g
*h^3*p*q + c^3*f^2*h^4*p*q - (p*q + q^2)*d^3*f^2*g^3*h)*b^3)*x)*log(d*x + c
))*log(h*x + g))/((d^3*g^4*h^3 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*
h^6)*a^3 - 3*(d^3*g^5*h^2 - 3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5
)*a^2*b + 3*(d^3*g^6*h - 3*c*d^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*a
*b^2 - (d^3*g^7 - 3*c*d^2*g^6*h + 3*c^2*d*g^5*h^2 - c^3*g^4*h^3)*b^3 + ((d^
3*g^3*h^4 - 3*c*d^2*g^2*h^5 + 3*c^2*d*g*h^6 - c^3*h^7)*a^3 - 3*(d^3*g^4*h^3
 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*h^6)*a^2*b + 3*(d^3*g^5*h^2 -
```

```
3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5)*a*b^2 - (d^3*g^6*h - 3*c*d
^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*b^3)*x))*r^2/(f^2*h) - 1/3*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)^3*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)^2}{{\left(g+h\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**4,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

```
[Out] -(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1+n)/b/c/(1+n)
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^n}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int x^n\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

Mathematica [**A**] time = 0.03, size = 42, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

fricas [**A**] time = 0.78, size = 56, normalized size = 1.33

$$-\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{bcn+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)*(b*log(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a)^n/(b*c*n + b*c)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b \log \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] -integrate((b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{n}}{c^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

sympy [A] time = 133.77, size = 94, normalized size = 2.24

$$\begin{cases} -\frac{a^n \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^n x & \text{for } c = 0 \\ -\frac{\begin{cases} \frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right) & \text{otherwise} \end{cases}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**n*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*n*x, Eq(c, 0)), (-Piecewise(((a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**(n
 + 1)/(n + 1), Ne(n, -1)), (log(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))), T
rue))/(b*c), True))
```

3.44 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

[Out] -1/4*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]

[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int x^3\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] -1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

fricas [B] time = 0.95, size = 101, normalized size = 2.73

$$-\frac{b^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4 + 4\,ab^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 6\,a^2b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 4\,a^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="fricas")
[Out] -1/4*(b^3*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^4 + 4*a*b^2*log(sqrt(-c*x + 1)/
sqrt(c*x + 1))^3 + 6*a^2*b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 4*a^3*log(
sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{3}}{-c^{2} x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^3/(-c^2*x^2+1),x)
```

```
[Out] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^3/(-c^2*x^2+1),x)
```

maxima [B] time = 1.53, size = 526, normalized size = 14.22

$$\frac{1}{2} b^{3}\left(\frac{\log \left(cx+1\right)}{c}-\frac{\log \left(cx-1\right)}{c}\right) \log \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{3}+\frac{3}{2} a b^{2}\left(\frac{\log \left(cx+1\right)}{c}-\frac{\log \left(cx-1\right)}{c}\right) \log \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}+\frac{3}{2} a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^3*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^3 + 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*
x + 1))^2 + 3/2*a^2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/
sqrt(c*x + 1)) + 1/64*(24*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + l
og(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2/c + 8*(log(c*x + 1)^3 -
3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1
)^3)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^4 - 4*log(c*x + 1)
^3*log(c*x - 1) + 6*log(c*x + 1)^2*log(c*x - 1)^2 - 4*log(c*x + 1)*log(c*x
- 1)^3 + log(c*x - 1)^4)/c)*b^3 + 1/8*a*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (
log(c*x + 1)^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1
)^2 - log(c*x - 1)^3)/c) + 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 3/8*
(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*a^2*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

sympy [A] time = 12.57, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^3 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^3 x & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*3*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c), Tr
ue))
```

3.45 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

```
[Out] -1/3*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int x^2\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

fricas [B] time = 0.83, size = 74, normalized size = 2.00

$$-\frac{b^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3\,ab\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3\,a^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -1/3*(b^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b*log(sqrt(-c*x + 1)/sq
rt(c*x + 1))^2 + 3*a^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^2/(-c^2*x^2+1),x)
```

```
[Out] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^2/(-c^2*x^2+1),x)
```

maxima [B] time = 1.25, size = 268, normalized size = 7.24

$$\frac{1}{2}b^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+ab\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{24}b^2\left(6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^2 + a*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + 1/24*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1
)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^3 - 3*log(c*x + 1)
^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1)^3)/c) + 1/2*
a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1
)*log(c*x - 1) + log(c*x - 1)^2)*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

sympy [A] time = 9.97, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^2 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^2 x & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*2*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c), Tr
ue))
```

3.46 $\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

```
[Out] -1/2*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2512, 2301}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\log(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

fricas [A] time = 0.89, size = 47, normalized size = 1.27

$$-\frac{b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,a\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="fricas")
```

```
[Out] -1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/c
```

giac [B] time = 0.21, size = 86, normalized size = 2.32

$$-\frac{b\log(cx+1)^2}{8\,c}+\frac{b\log(cx-1)^2}{8\,c}+\frac{1}{4}\left(\frac{b\log(cx+1)}{c}-\frac{b\log(cx-1)}{c}\right)\log(-cx+1)+\frac{a\log(cx+1)}{2\,c}-\frac{a\log(cx-1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="giac")
```

```
[Out] -1/8*b*log(c*x + 1)^2/c + 1/8*b*log(c*x - 1)^2/c + 1/4*(b*log(c*x + 1)/c -
b*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*a*log(c*x + 1)/c - 1/2*a*log(c*x - 1)
/c
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)/(-c^2*x^2+1),x)
```

```
[Out] int((b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)/(-c^2*x^2+1),x)
```

maxima [B] time = 1.02, size = 105, normalized size = 2.84

$$\frac{1}{2}\,b\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{2}\,a\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)+\frac{\left(\log\left(cx+1\right)^2-2\,\log\left(cx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="maxima")
```

```
[Out] 1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) +
 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

sympy [A] time = 9.30, size = 61, normalized size = 1.65

$$\begin{cases} -\frac{a\,\mathrm{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ ax & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**2/(2*b*c), True))
```

3.47 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

```
[Out] -ln(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/b/c
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 29}

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

Mathematica [**A**] time = 0.04, size = 34, normalized size = 1.00

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

fricas [**A**] time = 0.64, size = 30, normalized size = 0.88

$$-\frac{\log\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="fricas")
[Out] -log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

giac [**A**] time = 0.24, size = 31, normalized size = 0.91

$$-\frac{\log\left(-b\log\left(cx+1\right)+b\log\left(-cx+1\right)+2\,a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="giac")
```

```
[Out] -log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(b \ln \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a),x)
[Out] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a),x)
```

maxima [A] time = 1.76, size = 36, normalized size = 1.06

$$-\frac{\log\left(-\frac{b \log\left(c x+1\right)-b \log\left(-c x+1\right)-2 \, a}{2 \, b}\right)}{b c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="maxima")
[Out] -log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

sympy [A] time = 23.96, size = 53, normalized size = 1.56

$$\begin{cases} \frac{x}{a} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{- \frac{\log{\left(x - \frac{1}{c} \right)}}{2 c} + \frac{\log{\left(x + \frac{1}{c} \right)}}{2 c}}{a} & \text{for}\: b = 0 \\ \frac{x}{a} & \text{for}\: c = 0 \\ - \frac{\log{\left(\frac{a}{b} + \frac{\log{\left(- c x + 1 \right)}}{2} - \frac{\log{\left(c x + 1 \right)}}{2} \right)}}{b c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/c)/
(2*c))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (-log(a/b + log(-c*x + 1)/2 - log(c*x
 + 1)/2)/(b*c), True))
```

3.48 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

```
[Out] 1/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

Mathematica [**A**] time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

fricas [**A**] time = 0.65, size = 29, normalized size = 0.85

$$\frac{1}{b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="fricas")
```

```
[Out] 1/(b^2*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a*b*c)
```

giac [**A**] time = 0.22, size = 34, normalized size = 1.00

$$-\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="giac")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)\left(b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^2,x)
```

maxima [A] time = 1.84, size = 34, normalized size = 1.00

$$-\frac{2}{b^{2}c\log\left(cx+1\right)-b^{2}c\log\left(-cx+1\right)-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="maxima")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{2}\,\left(c^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

```
[Out] 1/2/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^3}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

fricas [A] time = 1.10, size = 59, normalized size = 1.59

$$\frac{1}{2\left(b^3c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="fricas")
[Out] 1/2/(b^3*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b^2*c*log(sqrt(-c*x +
1)/sqrt(c*x + 1)) + a^2*b*c)
```

giac [B] time = 0.33, size = 85, normalized size = 2.30

$$\frac{2}{b^3c\log(cx+1)^2-2\,b^3c\log(cx+1)\log(-cx+1)+b^3c\log(-cx+1)^2-4\,ab^2c\log(cx+1)+4\,ab^2c\log(-cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="giac")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 - 2*b^3*c*log(c*x + 1)*log(-c*x + 1) + b^3*c*log(-c
*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) + 4*a*b^2*c*log(-c*x + 1) + 4*a^2*b*c)
```

maple [**F**] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(b \ln \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^3,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))+a)^3,x)
```

maxima [**B**] time = 2.16, size = 80, normalized size = 2.16

$$\frac{2}{b^{3} c \log \left(c x+1\right)^{2}+b^{3} c \log \left(-c x+1\right)^{2}-4 \, a b^{2} c \log \left(c x+1\right)+4 \, a^{2} b c-2 \, {\left(b^{3} c \log \left(c x+1\right)-2 \, a b^{2} c\right)} \log \left(-c x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="maxima")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 + b^3*c*log(-c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) +
4*a^2*b*c - 2*(b^3*c*log(c*x + 1) - 2*a*b^2*c)*log(-c*x + 1))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^3\,\left(c^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3,x)
[Out] Timed out
```

3.50 $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=30

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/a
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2505}

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(2*a)
```

Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]}
, Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c -
 a*d)), x] /; FreeQ[h, x]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

Rubi steps

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

fricas [A] time = 0.61, size = 24, normalized size = 0.80

$$-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] -1/2*log(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/a
```

giac [B] time = 0.28, size = 58, normalized size = 1.93

$$\frac{1}{4}\left(\frac{\log\left(ax+1\right)}{a}-\frac{\log\left(ax-1\right)}{a}\right)\log\left(-ax+1\right)-\frac{\log\left(ax+1\right)^2}{8\,a}+\frac{\log\left(ax-1\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] 1/4*(log(a*x + 1)/a - log(a*x - 1)/a)*log(-a*x + 1) - 1/8*log(a*x + 1)^2/a
+ 1/8*log(a*x - 1)^2/a
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [B] time = 2.53, size = 83, normalized size = 2.77

$$\frac{1}{2}\left(\frac{\log\left(ax+1\right)}{a}-\frac{\log\left(ax-1\right)}{a}\right)\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)+\frac{\log\left(ax-1\right)^2}{8\,a}+\frac{\log\left(ax+1\right)^2-2\,\log\left(ax+1\right)\log\left(ax-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1
/8*log(a*x - 1)^2/a + 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{\ln\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [B] time = 6.28, size = 65, normalized size = 2.17

$$-\frac{\operatorname{atan}^2\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{a^2\sqrt{-\frac{1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -atan(x/sqrt(-1/a**2))**2/(2*a) - log(sqrt(-a*x + 1)/sqrt(a*x + 1))*atan(x/
sqrt(-1/a**2))/(a**2*sqrt(-1/a**2))
```

3.51 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx$

Optimal. Leaf size=410

$$\frac{\left(t\log\left(i(g+hx)^n\right)+s\right)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3hknt}-\frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nprt\text{Li}_3\left(\frac{b(g+}{bg-}\right.}{}$$

```
[Out] -1/3*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-1/3*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t+1/3*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)^2*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))^2*polylog
(2,d*(h*x+g)/(-c*h+d*g))/h/k+2*n*p*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,b*(h
*x+g)/(-a*h+b*g))/h/k+2*n*q*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,d*(h*x+g)/(
-c*h+d*g))/h/k-2*n^2*p*r*t^2*polylog(4,b*(h*x+g)/(-a*h+b*g))/h/k-2*n^2*q*r*
t^2*polylog(4,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A] time = 0.47, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2499, 2396, 2433, 2374, 2383, 6589}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{2n^2prt^2\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(g*
k + h*k*x),x]
```

```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^3)/(3*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
3)/(3*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^3)/(3*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g
+ h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2,
(d*(g + h*x))/(d*g - c*h)])/(h*k) + (2*n*p*r*t*(s + t*Log[i*(g + h*x)^n])*P
olyLog[3, (b*(g + h*x))/(b*g - a*h)])/(h*k) + (2*n*q*r*t*(s + t*Log[i*(g +
h*x)^n])*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k) - (2*n^2*p*r*t^2*Poly
Log[4, (b*(g + h*x))/(b*g - a*h)])/(h*k) - (2*n^2*q*r*t^2*PolyLog[4, (d*(g
+ h*x))/(d*g - c*h)])/(h*k)
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(51(g\right.\right.}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} -$$

Mathematica [B] time = 7.49, size = 22595, normalized size = 55.11

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^
2)/(g*k + h*k*x),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(t^2\log\left(\left(hx+g\right)^n i\right)^2+2\,st\log\left(\left(hx+g\right)^n i\right)+s^2\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="fricas")
```

```
[Out] integral((t^2*log((h*x + g)^n*i)^2 + 2*s*t*log((h*x + g)^n*i) + s^2)*log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)^{2}\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)
/(h*k*x + g*k), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(t\ln\left(i\left(hx+g\right)^{n}\right)+s\right)^{2}\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="maxima")
```

```
[Out] 1/3*((n^2*t^2*log(h*x + g)^3 + 3*t^2*log(h*x + g)*log((h*x + g)^n)^2 - 3*(n
*t^2*log(i) + n*s*t)*log(h*x + g)^2 + 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)
*log(h*x + g) - 3*(n*t^2*log(h*x + g)^2 - 2*(t^2*log(i) + s*t)*log(h*x + g)
)*log((h*x + g)^n))*log(((b*x + a)^p)^r) + (n^2*t^2*log(h*x + g)^3 + 3*t^2*
log(h*x + g)*log((h*x + g)^n)^2 - 3*(n*t^2*log(i) + n*s*t)*log(h*x + g)^2 +
```

```
 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)*log(h*x + g) - 3*(n*t^2*log(h*x + g)
^2 - 2*(t^2*log(i) + s*t)*log(h*x + g))*log((h*x + g)^n))*log(((d*x + c)^q)
^r))/(h*k) - integrate(-1/3*(3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e
) + (r*t^2*log(i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f))*b*d*x^2 - ((p*r + q
*r)*b*d*h*n^2*t^2*x^2 + b*c*g*n^2*p*r*t^2 + a*d*g*n^2*q*r*t^2 + (a*d*h*n^2*
q*r*t^2 + (c*h*n^2*p*r*t^2 + (p*r + q*r)*d*g*n^2*t^2)*b)*x)*log(h*x + g)^3
+ 3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 + 2*r*s
*t*log(i) + r*s^2)*h*log(f))*a*c + 3*(((p*r + q*r)*n*t^2*log(i) + (p*r*s +
q*r*s)*n*t)*b*d*h*x^2 + (n*p*r*t^2*log(i) + n*p*r*s*t)*b*c*g + (n*q*r*t^2*l
og(i) + n*q*r*s*t)*a*d*g + ((n*q*r*t^2*log(i) + n*q*r*s*t)*a*d*h + (((p*r +
 q*r)*n*t^2*log(i) + (p*r*s + q*r*s)*n*t)*d*g + (n*p*r*t^2*log(i) + n*p*r*s
*t)*c*h)*b)*x)*log(h*x + g)^2 + 3*((h*r*t^2*log(f) + h*t^2*log(e))*b*d*x^2
+ (h*r*t^2*log(f) + h*t^2*log(e))*a*c + ((h*r*t^2*log(f) + h*t^2*log(e))*b*
c + (h*r*t^2*log(f) + h*t^2*log(e))*a*d)*x - ((p*r + q*r)*b*d*h*t^2*x^2 + b
*c*g*p*r*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*
d*g*t^2)*b)*x)*log(h*x + g))*log((h*x + g)^n)^2 + 3*(((t^2*log(i)^2 + 2*s*t
*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f
))*b*c + ((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 +
2*r*s*t*log(i) + r*s^2)*h*log(f))*a*d)*x - 3*(((p*r + q*r)*t^2*log(i)^2 + p
*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log(i))*b*d*h*x^2 + (p*r*t^2*log(i)^
2 + 2*p*r*s*t*log(i) + p*r*s^2)*b*c*g + (q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i
) + q*r*s^2)*a*d*g + ((q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d*h
 + (((p*r + q*r)*t^2*log(i)^2 + p*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log
(i))*d*g + (p*r*t^2*log(i)^2 + 2*p*r*s*t*log(i) + p*r*s^2)*c*h)*b)*x)*log(h
*x + g) + 3*(2*((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(
f))*b*d*x^2 + 2*((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log
(f))*a*c + ((p*r + q*r)*b*d*h*n*t^2*x^2 + b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2
 + (a*d*h*n*q*r*t^2 + (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b)*x)*log(h*x
 + g)^2 + 2*(((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(f)
)*b*c + ((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(f))*a*d
)*x - 2*(((p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*b*d*h*x^2 + (p*r*t^2*
log(i) + p*r*s*t)*b*c*g + (q*r*t^2*log(i) + q*r*s*t)*a*d*g + ((q*r*t^2*log(
i) + q*r*s*t)*a*d*h + (((p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*d*g + (
p*r*t^2*log(i) + p*r*s*t)*c*h)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h
^2*k*x^3 + a*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k
 + (d*g*h*k + c*h^2*k)*a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,{\left(s+t\,\ln\left(i\,{\left(g+h\,x\right)}^n\right)\right)}^2}{g\,k+h\,k\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*k
*x+g*k),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$

Optimal. Leaf size=306

$$\frac{\left(t\log\left(i(g+hx)^n\right)+s\right)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2hknt}-\frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right.}{}$$

```
[Out] -1/2*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-1/2*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t+1/2*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))*polylog(2,d
*(h*x+g)/(-c*h+d*g))/h/k+n*p*r*t*polylog(3,b*(h*x+g)/(-a*h+b*g))/h/k+n*q*r*
t*polylog(3,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A] time = 0.29, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2499, 2396, 2433, 2374, 6589}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}+\frac{nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+\right.\right.}{hk}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))/(g*k
+ h*k*x),x]
```

```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g +
h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(
g + h*x))/(d*g - c*h)])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g - a*
h)])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+hx)^n\right)\right)}{gk+hkx}\,dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+\right.\right.}{2hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} -$$

Mathematica [A] time = 1.54, size = 436, normalized size = 1.42

$$\frac{2t\log(g+hx)\log\left(i(g+hx)^n\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - nt\log^2(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))
/(g*k + h*k*x),x]

[Out] (-2*p*r*s*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] - 2*q*r*s*Log[(h*(
c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] + 2*s*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]*Log[g + h*x] + n*p*r*t*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x
]^2 + n*q*r*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2 - n*t*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 - 2*p*r*t*Log[(h*(a + b*x))/(
-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*q*r*t*Log[(h*(c + d*x))/
(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 2*t*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*p*r*(s + t*Log[i*(g + h
*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] - 2*q*r*(s + t*Log[i*(g + h*x
)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] + 2*n*p*r*t*PolyLog[3, (b*(g +
```

```
h*x))/(b*g - a*h)] + 2*n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(2*h*
k)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="fricas")
```

```
[Out] integral((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h
*k*x + g*k), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(
h*k*x + g*k), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\frac{\left(t\ln\left(i\left(hx+g\right)^{n}\right)+s\right)\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(t*ln(i*(h*x+g)^n)+s)/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(t*ln(i*(h*x+g)^n)+s)/(h*k*x+g*k),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(nt\log\left(hx+g\right)^{2}-2\,t\log\left(hx+g\right)\log\left(\left(hx+g\right)^{n}\right)-2\left(t\log(i)+s\right)\log\left(hx+g\right)\right)\log\left(\left((bx+a)^{p}\right)^{r}\right)+\left(nt\log\right.}{2\,hk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="maxima")
```

```
[Out] -1/2*((n*t*log(h*x + g)^2 - 2*t*log(h*x + g)*log((h*x + g)^n) - 2*(t*log(i)
 + s)*log(h*x + g))*log(((b*x + a)^p)^r) + (n*t*log(h*x + g)^2 - 2*t*log(h*
x + g)*log((h*x + g)^n) - 2*(t*log(i) + s)*log(h*x + g))*log(((d*x + c)^q)^
r))/(h*k) - integrate(-1/2*(2*((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)
*h*log(f))*b*d*x^2 + 2*((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(
f))*a*c + ((p*r + q*r)*b*d*h*n*t*x^2 + b*c*g*n*p*r*t + a*d*g*n*q*r*t + (a*d
*h*n*q*r*t + (c*h*n*p*r*t + (p*r + q*r)*d*g*n*t)*b)*x)*log(h*x + g)^2 + 2*(
((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*b*c + ((t*log(i) +
s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*a*d)*x - 2*((p*r*s + q*r*s + (p*
r + q*r)*t*log(i))*b*d*h*x^2 + (p*r*t*log(i) + p*r*s)*b*c*g + (q*r*t*log(i)
 + q*r*s)*a*d*g + ((q*r*t*log(i) + q*r*s)*a*d*h + ((p*r*s + q*r*s + (p*r +
q*r)*t*log(i))*d*g + (p*r*t*log(i) + p*r*s)*c*h)*b)*x)*log(h*x + g) + 2*((h
*r*t*log(f) + h*t*log(e))*b*d*x^2 + (h*r*t*log(f) + h*t*log(e))*a*c + ((h*r
*t*log(f) + h*t*log(e))*b*c + (h*r*t*log(f) + h*t*log(e))*a*d)*x - ((p*r +
q*r)*b*d*h*t*x^2 + b*c*g*p*r*t + a*d*g*q*r*t + (a*d*h*q*r*t + (c*h*p*r*t +
(p*r + q*r)*d*g*t)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a
*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k
+ c*h^2*k)*a)*x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(s+t\,\ln\left(i\left(g+h\,x\right)^n\right)\right)}{g\,k+h\,k\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))/(h*k*x+
g*k),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx}\,dx$

Optimal. Leaf size=172

$$\frac{\log(gk+hkx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}-\frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{pr\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{hk}-\frac{qr\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk}-\cdots$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*k*x+g*k)/h/k-q*r*ln(-h*(d*x+c)/(-c*h+d*
g))*ln(h*k*x+g*k)/h/k+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*k*x+g*k)/h/k-p*r
*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h/
k
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2494, 2394, 2393, 2391}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}-\frac{pr\log(gk+hkx\cdots}{h\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/(h*k)) - (q*r*Lo
g[-((h*(c + d*x))/(d*g - c*h))]*Log[g*k + h*k*x])/(h*k) + (Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (p*r*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/(h*k) - (q*r*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx}\,dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(gk+hkx)}{hk} - \frac{(bpr)\int\frac{\log(gk+hkx)}{a+bx}\,dx}{hk} - \frac{(d$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log$$

Mathematica [A] time = 0.09, size = 166, normalized size = 0.97

$$\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + pr\mathrm{Li}_2\left(\frac{h(a+bx)}{ah-bg}\right) - pr\log(a+bx)\log(g+hx) + pr\log(a+bx)\log\left(\frac{b}{}\right.}{hk}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/(h*k)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

maxima [A] time = 1.20, size = 204, normalized size = 1.19

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{hk}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{hk}\right)r}{f}-\frac{\left(fp\log\left(bx+a\right)+fq\log\left(dx+c\right)\right)r}{fhk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="maxima
")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) +
dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) + f*q
*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x + a)
^p*(d*x + c)^q*f)^r*e)/(h*k)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{g\,k+h\,k\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx}{k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(g + h*x), x)/k
```

3.54 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$

Optimal. Leaf size=51

$$\text{Int}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)},x\right)$$

```
[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^
n)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])), x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx$$

Mathematica [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hksx+gks+\left(hktx+gkt\right)\log\left(\left(hx+g\right)^n i\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s*x + g*k*s + (h*k*t*x +
 g*k*t)*log((h*x + g)^n*i)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{\left(hkx+gk\right)\left(t\log\left(\left(hx+g\right)^n i\right)+s\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

maple [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{\left(hkx+gk\right)\left(t\ln\left(i\left(hx+g\right)^n\right)+s\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(t*ln(i*(h*x+g)^n)+s),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(t*ln(i*(h*x+g)^n)+s),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{\left(hkx+gk\right)\left(t\log\left(\left(hx+g\right)^n i\right)+s\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{\left(g\,k+h\,k\,x\right)\,\left(s+t\,\ln\left(i\left(g+h\,x\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n)),x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$

Optimal. Leaf size=51

$$\mathrm{Int}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)^2},x\right)$$

[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h*x)^n])^2),x]

[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h*x)^n])^2), x]

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx$$

Mathematica [A] time = 2.87, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hks^2x+gks^2+\left(hkt^2x+gkt^2\right)\log\left(\left(hx+g\right)^n i\right)^2+2\left(hkstx+gkst\right)\log\left(\left(hx+g\right)^n i\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s^2*x + g*k*s^2 + (h*k*t
^2*x + g*k*t^2)*log((h*x + g)^n*i)^2 + 2*(h*k*s*t*x + g*k*s*t)*log((h*x + g
)^n*i)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{\left(hkx+gk\right)\left(t\log\left(\left(hx+g\right)^n i\right)+s\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)^2), x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{\left(hkx+gk\right)\left(t\ln\left(i\left(hx+g\right)^n\right)+s\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(t*ln(i*(h*x+g)^n)+s)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(t*ln(i*(h*x+g)^n)+s)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{r\log(f)+\log\left(\left((bx+a)^p\right)^r\right)+\log\left(\left((dx+c)^q\right)^r\right)+\log(e)}{hknt^2\log\left(\left(hx+g\right)^n\right)+\left(knt^2\log(i)+knst\right)h}+\int\frac{}{\left(knt^2\log(i)+knst\right)bdhx^2+\left(knt^2\log(i)+kns\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="maxima")
```

```
[Out] -(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(h*k*n*t
^2*log((h*x + g)^n) + (k*n*t^2*log(i) + k*n*s*t)*h) + integrate((b*c*p*r +
a*d*q*r + (p*r + q*r)*b*d*x)/((k*n*t^2*log(i) + k*n*s*t)*b*d*h*x^2 + (k*n*t
^2*log(i) + k*n*s*t)*a*c*h + ((k*n*t^2*log(i) + k*n*s*t)*b*c*h + (k*n*t^2*l
og(i) + k*n*s*t)*a*d*h)*x + (b*d*h*k*n*t^2*x^2 + a*c*h*k*n*t^2 + (b*c*h*k*n
*t^2 + a*d*h*k*n*t^2)*x)*log((h*x + g)^n)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{\left(g\,k+h\,k\,x\right)\,\left(s+t\,\ln\left(i\left(g+h\,x\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n))**2,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=328

$$\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu}-6prt^2u^2\text{Li}_4\left(-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-pr\text{Li}_2\left(-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right.$$

```
[Out] -1/4*p*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+b*x/a)/t/u+1/4*ln(i*(j*(h*x)^t)^u)^4*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/4*q*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^3*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^3*
polylog(2,-d*x/c)+3*p*r*t*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-b*x/a)+3*q*r*t
*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-d*x/c)-6*p*r*t^2*u^2*ln(i*(j*(h*x)^t)^u
)*polylog(4,-b*x/a)-6*q*r*t^2*u^2*ln(i*(j*(h*x)^t)^u)*polylog(4,-d*x/c)+6*p
*r*t^3*u^3*polylog(5,-b*x/a)+6*q*r*t^3*u^3*polylog(5,-d*x/c)
```

Rubi [A] time = 1.25, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-6prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right)+3prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(4*t*u) + (Log[i*(j*(h*x)^t)
^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/c)] + 3*p*r*t*u*L
og[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)] + 3*q*r*t*u*Log[i*(j*(h*x)^t)^
u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4,
 -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)] +
6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(56\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log^3\left(56j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{x}\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^3\left(56h^{tu}j^u x^{tu}\right)\log\left(e\left(f(a+bx)^p(\right.\right.}{x}\right.\right.$$

$$= \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu} - \text{Sul}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right.\right.}{}$$

Mathematica [B] time = 1.84, size = 1241, normalized size = 3.78

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a +
 b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] +
```

```
 3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*Log
[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2*Log
[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[a
+ b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*
x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] + q*r*t^3*u
^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c + d*x] - 3
*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t^
2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*Log[x]*Log[h
*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*
x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] +
q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*Log[x]*Log[h*x]^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x]^4*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)
^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[h*x]^3*Log[i*(j*(h
*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*Log[x]*Log[h*x]*Log[
i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Lo
g[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (d*x)/c])/4 - q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (d*x)/c] + (3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*
x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c] - p*r*Log[i
*(j*(h*x)^t)^u]^3*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyL
og[2, -((d*x)/c)] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)]
 + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*
Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^
t)^u]*PolyLog[4, -((d*x)/c)] + 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r
*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log\left(\left((hx)^{t}j\right)^{u}i\right)^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

maple [F] time = 2.92, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\ln\left(i\left(j\,(hx)^t\right)^u\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] -1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h) + t^2*u^3*log(j) + t^2*u^2*log(i
))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t^3*u^3*log(h)^2 + t*u^3*log(j)^
2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2 + 2*(t^2*u^3*log(j) + t^2*u^2*log(
i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))
*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)
*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h
)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*
log(h))*log(x) - 4*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t
*u*log(i))*log(x)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j
) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x))*log((x^t)^u))*
log(((b*x + a)^p)^r) - 1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h) + t^2*u^3*
log(j) + t^2*u^2*log(i))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t^3*u^3*lo
g(h)^2 + t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2 + 2*(t^2*u^3
```

```
*log(j) + t^2*u^2*log(i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(t*u*log(h
) + u*log(j) + log(i))*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*log(h)^3 + u^3*l
og(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) +
 t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*l
og(j) + t*u*log(i)^2)*log(h))*log(x) - 4*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log
(h) + t*u^2*log(j) + t*u*log(i))*log(x)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j
)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*
log(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/4*(4*((t^3*u^3*lo
g(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^
2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*
t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) + (r*t^3*u^3*log(h)^3 +
r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log(i)^2*log(j) + r*log(i)
^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^2 + 3*(r*t*u^3*log(j)^2
 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))*log(f))*b*d*x^2 + ((p*
r*t^3*u^3 + q*r*t^3*u^3)*b*d*x^2 + (b*c*p*r*t^3*u^3 + a*d*q*r*t^3*u^3)*x)*l
og(x)^4 + 4*((r*log(f) + log(e))*b*d*x^2 + (r*log(f) + log(e))*a*c + ((r*lo
g(f) + log(e))*b*c + (r*log(f) + log(e))*a*d)*x - ((p*r + q*r)*b*d*x^2 + (b
*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^3 - 4*(((p*r*t^3*u^3 + q*r*t^3*u^
3)*log(h) + (p*r*t^2*u^2 + q*r*t^2*u^2)*log(i) + (p*r*t^2*u^3 + q*r*t^2*u^3
)*log(j))*b*d*x^2 + ((p*r*t^3*u^3*log(h) + p*r*t^2*u^3*log(j) + p*r*t^2*u^2
*log(i))*b*c + (q*r*t^3*u^3*log(h) + q*r*t^2*u^3*log(j) + q*r*t^2*u^2*log(i
))*a*d)*x)*log(x)^3 + 4*((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*lo
g(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2
 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log
(h))*log(e) + (r*t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^
2 + 3*r*u*log(i)^2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*lo
g(i))*log(h)^2 + 3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(
i)^2)*log(h))*log(f))*a*c + 6*(((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h)^2 + (p*r
*t*u + q*r*t*u)*log(i)^2 + 2*(p*r*t*u^2 + q*r*t*u^2)*log(i)*log(j) + (p*r*t
*u^3 + q*r*t*u^3)*log(j)^2 + 2*((p*r*t^2*u^2 + q*r*t^2*u^2)*log(i) + (p*r*t
^2*u^3 + q*r*t^2*u^3)*log(j))*log(h))*b*d*x^2 + ((p*r*t^3*u^3*log(h)^2 + p*
r*t*u^3*log(j)^2 + 2*p*r*t*u^2*log(i)*log(j) + p*r*t*u*log(i)^2 + 2*(p*r*t^
2*u^3*log(j) + p*r*t^2*u^2*log(i))*log(h))*b*c + (q*r*t^3*u^3*log(h)^2 + q*
r*t*u^3*log(j)^2 + 2*q*r*t*u^2*log(i)*log(j) + q*r*t*u*log(i)^2 + 2*(q*r*t^
2*u^3*log(j) + q*r*t^2*u^2*log(i))*log(h))*a*d)*x)*log(x)^2 + 6*(2*((t*u*lo
g(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*l
og(f))*b*d*x^2 + 2*((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h)
 + r*u*log(j) + r*log(i))*log(f))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c
*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((t*u*log(h) + u*log(j) + log(i))*
log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*b*c + ((t*u*log(h)
+ u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f)
)*a*d)*x - 2*(((p*r*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q
*r*u)*log(j))*b*d*x^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c +
 (q*r*t*u*log(h) + q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))*log((x^t)^u)^
2 + 4*(((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(
```

```
i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(
t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) + (r*
t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log(i)^
2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^2 +
3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))*log
(f))*b*c + ((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*
log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 +
 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) +
 (r*t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log
(i)^2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^
2 + 3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))
*log(f))*a*d)*x - 4*(((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h)^3 + (p*r + q*r)*lo
g(i)^3 + 3*(p*r*u + q*r*u)*log(i)^2*log(j) + 3*(p*r*u^2 + q*r*u^2)*log(i)*l
og(j)^2 + (p*r*u^3 + q*r*u^3)*log(j)^3 + 3*((p*r*t^2*u^2 + q*r*t^2*u^2)*log
(i) + (p*r*t^2*u^3 + q*r*t^2*u^3)*log(j))*log(h)^2 + 3*((p*r*t*u + q*r*t*u)
*log(i)^2 + 2*(p*r*t*u^2 + q*r*t*u^2)*log(i)*log(j) + (p*r*t*u^3 + q*r*t*u^
3)*log(j)^2)*log(h))*b*d*x^2 + ((p*r*t^3*u^3*log(h)^3 + p*r*u^3*log(j)^3 +
3*p*r*u^2*log(i)*log(j)^2 + 3*p*r*u*log(i)^2*log(j) + p*r*log(i)^3 + 3*(p*r
*t^2*u^3*log(j) + p*r*t^2*u^2*log(i))*log(h)^2 + 3*(p*r*t*u^3*log(j)^2 + 2*
p*r*t*u^2*log(i)*log(j) + p*r*t*u*log(i)^2)*log(h))*b*c + (q*r*t^3*u^3*log(
h)^3 + q*r*u^3*log(j)^3 + 3*q*r*u^2*log(i)*log(j)^2 + 3*q*r*u*log(i)^2*log(
j) + q*r*log(i)^3 + 3*(q*r*t^2*u^3*log(j) + q*r*t^2*u^2*log(i))*log(h)^2 +
3*(q*r*t*u^3*log(j)^2 + 2*q*r*t*u^2*log(i)*log(j) + q*r*t*u*log(i)^2)*log(h
))*a*d)*x)*log(x) + 4*(3*((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log
(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r*t^2*u^2*
log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*l
og(j) + r*t*u*log(i))*log(h))*log(f))*b*d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^2
)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((t^2*u^2*l
og(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*
log(h) + log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*lo
g(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f)
)*a*c + 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*u + q*r*t*u)*log(i)
 + (p*r*t*u^2 + q*r*t*u^2)*log(j))*b*d*x^2 + ((p*r*t^2*u^2*log(h) + p*r*t*u
^2*log(j) + p*r*t*u*log(i))*b*c + (q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j) +
q*r*t*u*log(i))*a*d)*x)*log(x)^2 + 3*(((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2
*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e)
+ (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 +
 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*c + ((t^2*u^2*log(h)^2
 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h)
+ log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*lo
g(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*a*d)*
x - 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h)^2 + (p*r + q*r)*log(i)^2 + 2*(p*
r*u + q*r*u)*log(i)*log(j) + (p*r*u^2 + q*r*u^2)*log(j)^2 + 2*((p*r*t*u + q
*r*t*u)*log(i) + (p*r*t*u^2 + q*r*t*u^2)*log(j))*log(h))*b*d*x^2 + ((p*r*t^
2*u^2*log(h)^2 + p*r*u^2*log(j)^2 + 2*p*r*u*log(i)*log(j) + p*r*log(i)^2 +
```

```
2*(p*r*t*u^2*log(j) + p*r*t*u*log(i))*log(h))*b*c + (q*r*t^2*u^2*log(h)^2 +
 q*r*u^2*log(j)^2 + 2*q*r*u*log(i)*log(j) + q*r*log(i)^2 + 2*(q*r*t*u^2*log
(j) + q*r*t*u*log(i))*log(h))*a*d)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*
x + (b*c + a*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\left(j\,(h\,x)^t\right)^u\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x,x)
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Timed out
```

3.57 $\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=262

$$\frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}-pr\text{Li}_2\left(-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)+2prtu\text{Li}_3\left(-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)$$

```
[Out] -1/3*p*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+b*x/a)/t/u+1/3*ln(i*(j*(h*x)^t)^u)^3*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/3*q*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^2*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^2*
polylog(2,-d*x/c)+2*p*r*t*u*ln(i*(j*(h*x)^t)^u)*polylog(3,-b*x/a)+2*q*r*t*u
*ln(i*(j*(h*x)^t)^u)*polylog(3,-d*x/c)-2*p*r*t^2*u^2*polylog(4,-b*x/a)-2*q*
r*t^2*u^2*polylog(4,-d*x/c)
```

Rubi [A] time = 0.91, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)+2prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-2prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)-qr$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/(3*t*u) + (Log[i*(j*(h*x)^t)
^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^3*Log[1 + (d*x)/c])/(3*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*L
og[i*(j*(h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]
*PolyLog[3, -((d*x)/c)] - 2*p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*
u^2*PolyLog[4, -((d*x)/c)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(57\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log^2\left(57j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{x}\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^2\left(57h^{tu}j^u x^{tu}\right)\log\left(e\left(f(a+bx)^p(\right.\right.}{x}\right.\right.$$

$$= \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu} - \text{Sul}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right.\right.}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right.\right.}{}$$

Mathematica [B] time = 0.88, size = 839, normalized size = 3.20

$$prt^2u^2\log(a+bx)\log^3(hx)-\frac{1}{3}prt^2u^2\log\left(\frac{bx}{a}+1\right)\log^3(hx)+qrt^2u^2\log(c+dx)\log^3(hx)-\frac{2}{3}t^2u^2\log\left(e\left(f(a+bx)^p(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]

[Out] -(p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[a + b*x]) + p*r*t^2*u^2*Log[h*x]^3*Log[
a + b*x] + 2*p*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - 2*
p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - p*r*Log[x]*Log[i*(j*
(h*x)^t)^u]^2*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*
x] - (p*r*t^2*u^2*Log[h*x]^3*Log[1 + (b*x)/a])/3 + p*r*t*u*Log[h*x]^2*Log[i
```

```
*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[
1 + (b*x)/a] - q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[c + d*x] + q*r*t^2*u^2*Log
[h*x]^3*Log[c + d*x] + 2*q*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c
 + d*x] - 2*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - q*r*Log[
x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^
2*Log[c + d*x] + t^2*u^2*Log[x]*Log[h*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - (2*t^2*u^2*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - 2*t*
u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (q*
r*t^2*u^2*Log[h*x]^3*Log[1 + (d*x)/c])/3 + q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x
)^t)^u]*Log[1 + (d*x)/c] - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x
)/c] - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*
x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[
3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((d*x)/c)] - 2*
p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*u^2*PolyLog[4, -((d*x)/c)]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

maple [F] time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\ln\left(i\left(j\,(hx)^t\right)^u\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] 1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*log(
x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*l
og(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x) - 3*
(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x))*log((x^t)^u))*lo
g(((b*x + a)^p)^r) + 1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(
j) + t*u*log(i))*log(x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(h)^2 +
 u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) +
log(i)^2)*log(x) - 3*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log
(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/3*(3*((t^2*u^2*log(h
)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(
h) + log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)
*log(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*
d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t
^2*u^2)*x)*log(x)^3 + 3*((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(
j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r*t^2*u^2*l
og(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*lo
g(j) + r*t*u*log(i))*log(h))*log(f))*a*c + 3*((r*log(f) + log(e))*b*d*x^2 +
 (r*log(f) + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*log(f) + log(e))*a
*d)*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^
2 + 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*u + q*r*t*u)*log(i) + (
p*r*t*u^2 + q*r*t*u^2)*log(j))*b*d*x^2 + ((p*r*t^2*u^2*log(h) + p*r*t*u^2*l
og(j) + p*r*t*u*log(i))*b*c + (q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j) + q*r*
t*u*log(i))*a*d)*x)*log(x)^2 + 3*(((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*l
og(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r
*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(
r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*c + ((t^2*u^2*log(h)^2 + u
^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + lo
g(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j)
 + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*a*d)*x -
3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h)^2 + (p*r + q*r)*log(i)^2 + 2*(p*r*u
+ q*r*u)*log(i)*log(j) + (p*r*u^2 + q*r*u^2)*log(j)^2 + 2*((p*r*t*u + q*r*t
```

```
*u)*log(i) + (p*r*t*u^2 + q*r*t*u^2)*log(j))*log(h))*b*d*x^2 + ((p*r*t^2*u^
2*log(h)^2 + p*r*u^2*log(j)^2 + 2*p*r*u*log(i)*log(j) + p*r*log(i)^2 + 2*(p
*r*t*u^2*log(j) + p*r*t*u*log(i))*log(h))*b*c + (q*r*t^2*u^2*log(h)^2 + q*r
*u^2*log(j)^2 + 2*q*r*u*log(i)*log(j) + q*r*log(i)^2 + 2*(q*r*t*u^2*log(j)
+ q*r*t*u*log(i))*log(h))*a*d)*x)*log(x) + 3*(2*((t*u*log(h) + u*log(j) + l
og(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*b*d*x^2 + 2*
((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*l
og(i))*log(f))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*
t*u)*x)*log(x)^2 + 2*(((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log
(h) + r*u*log(j) + r*log(i))*log(f))*b*c + ((t*u*log(h) + u*log(j) + log(i)
)*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*a*d)*x - 2*(((p*r
*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q*r*u)*log(j))*b*d*x
^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c + (q*r*t*u*log(h) +
q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*x +
 (b*c + a*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\left(j\,(h\,x)^t\right)^u\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x,x)
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Timed out
```

3.58 $\int \frac{\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=194

$$\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}-pr\text{Li}_2\left(-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-\frac{pr\log\left(\frac{bx}{a}+1\right)\log^2\left(i\left(j(hx)^t\right)^u\right)}{2tu}+$$

```
[Out] -1/2*p*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+b*x/a)/t/u+1/2*ln(i*(j*(h*x)^t)^u)^2*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/2*q*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)*poly
log(2,-d*x/c)+p*r*t*u*polylog(3,-b*x/a)+q*r*t*u*polylog(3,-d*x/c)
```

Rubi [A] time = 0.60, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2499, 2317, 2374, 6589, 2445}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)+prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)+qrtu\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(2*t*u) + (Log[i*(j*(h*x)^t)
^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b
*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog
[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[n] && !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(58\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log\left(58j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right.}{x}\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log\left(58h^{tu}j^ux^{tu}\right)\log\left(e\left(f(a+bx)^p(c+\right.\right.}{x}\right.\right.$$

$$= \frac{\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} - \text{Sub}$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right.}{}$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right.}{}$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right.}{}$$

Mathematica [B] time = 0.43, size = 451, normalized size = 2.32

$$\log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\frac{1}{2}tu\log^2(hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-tu\log(x)\log($$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] p*r*t*u*Log[x]*Log[h*x]*Log[a + b*x] - p*r*t*u*Log[h*x]^2*Log[a + b*x] - p*
r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)
^u]*Log[a + b*x] + (p*r*t*u*Log[h*x]^2*Log[1 + (b*x)/a])/2 - p*r*Log[h*x]*L
og[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] + q*r*t*u*Log[x]*Log[h*x]*Log[c + d*x]
 - q*r*t*u*Log[h*x]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[c
+ d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - t*u*Log[x]*Log[h*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (t*u*Log[h*x]^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/2 + Log[x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] + (q*r*t*u*Log[h*x]^2*Log[1 + (d*x)/c])/2 - q*r*Log[h*x]*Log
[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -(
```

```
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyL
og[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log\left(\left((hx)^{t}j\right)^{u}i\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log\left(\left((hx)^{t}j\right)^{u}i\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

maple [F] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(tu\log(x)^{2}-2\left(tu\log(h)+u\log(j)+\log(i)\right)\log(x)-2\,\log(x)\log\left(\left(x^{t}\right)^{u}\right)\right)\log\left(\left((bx+a)^{p}\right)^{r}\right)-\frac{1}{2}\left(tu\log(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="maxima")
```

```
[Out] -1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x) - 2*log(x)*l
og((x^t)^u))*log(((b*x + a)^p)^r) - 1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u*l
og(j) + log(i))*log(x) - 2*log(x)*log((x^t)^u))*log(((d*x + c)^q)^r) - inte
grate(-1/2*(2*((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*
u*log(j) + r*log(i))*log(f))*b*d*x^2 + 2*((t*u*log(h) + u*log(j) + log(i))*
log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*a*c + ((p*r*t*u + q
*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((t*u*log(h)
 + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f
))*b*c + ((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log
(j) + r*log(i))*log(f))*a*d)*x + 2*((r*log(f) + log(e))*b*d*x^2 + (r*log(f)
 + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*log(f) + log(e))*a*d)*x - ((
p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u) - 2*(((p*r
*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q*r*u)*log(j))*b*d*x
^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c + (q*r*t*u*log(h) +
q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))/(b*d*x^3 + a*c*x + (b*c + a*d)*x
^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\left(j\,(h\,x)^t\right)^u\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x,x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=81

$$\log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\text{Li}_2\left(-\frac{bx}{a}\right)-pr\log(x)\log\left(\frac{bx}{a}+1\right)-qr\text{Li}_2\left(-\frac{dx}{c}\right)-qr\log(x)\log\left(\frac{dx}{c}+1\right)$$

```
[Out] -p*r*ln(x)*ln(1+b*x/a)+ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-q*r*ln(x)*ln(1
+d*x/c)-p*r*polylog(2,-b*x/a)-q*r*polylog(2,-d*x/c)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2494, 2317, 2391}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(x)\log\left(\frac{bx}{a}+1\right)-qr\log(x)\log\left(\frac{dx}{c}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] -(p*r*Log[x]*Log[1 + (b*x)/a]) + Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] - q*r*Log[x]*Log[1 + (d*x)/c] - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[
2, -((d*x)/c)]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - (bpr)\int \frac{\log(x)}{a+bx}\,dx - (dqr)\int \frac{\log(x}{c+d}$$

$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr\log(x$$

$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr\log(x$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.96

$$\log(x)\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr\log\left(\frac{bx}{a}+1\right) - qr\log\left(\frac{dx}{c}+1\right)\right) - pr\text{Li}_2\left(-\frac{bx}{a}\right) - qr\text{Li}_2\left(-\frac{dx}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q*
r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)/c
)]
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

maxima [A] time = 1.14, size = 126, normalized size = 1.56

$$-\frac{\left(fp\log\left(bx+a\right)+fq\log\left(dx+c\right)\right)r\log(x)}{f}+\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log(x)+\frac{\left(\left(\log\left(bx+a\right)\log\left(-\frac{bx+a}{a}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="maxima")
```

```
[Out] -(f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(x)/f + log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)*log(x) + ((log(b*x + a)*log(-(b*x + a)/a + 1) + dilog((b*x +
a)/a))*f*p + (log(d*x + c)*log(-(d*x + c)/c + 1) + dilog((d*x + c)/c))*f*q)
*r/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\,(a+bx)^{p}\,(c+dx)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/x, x)
```

3.60 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x
]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

maple [A] time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{x\ln\left(i\left(j\,(hx)^t\right)^u\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{x\,\ln\left(i\left(j\,(h\,x)^t\right)^u\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Rubi [A] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
 x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A] time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x
]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
x]
```

fricas [A] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2),
x)
```

maple [A] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{x\ln\left(i\left(j\,(hx)^t\right)^u\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{r\log(f)+\log\left(\left((bx+a)^p\right)^r\right)+\log\left(\left((dx+c)^q\right)^r\right)+\log(e)}{t^2u^2\log(h)+tu^2\log(j)+tu\log(i)+tu\log\left(\left(x^t\right)^u\right)}+\int\frac{}{\left(t^2u^2\log(h)+tu^2\log(j)+tu\log(i)\right)bdx^2+\left(t^2u\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="maxima")
```

```
[Out] -(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(t^2*u^2
*log(h) + t*u^2*log(j) + t*u*log(i) + t*u*log((x^t)^u)) + integrate((b*c*p*
r + a*d*q*r + (p*r + q*r)*b*d*x)/((t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(
i))*b*d*x^2 + (t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*a*c + ((t^2*u^2*
log(h) + t*u^2*log(j) + t*u*log(i))*b*c + (t^2*u^2*log(h) + t*u^2*log(j) +
t*u*log(i))*a*d)*x + (b*d*t*u*x^2 + a*c*t*u + (b*c*t*u + a*d*t*u)*x)*log((x
^t)^u)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{x\ln\left(i\left(j\,(h\,x)^t\right)^u\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u)**2,x)
```

```
[Out] Timed out
```

3.62 $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)\log^3\left(\frac{a+bx}{x(bc-ad)}\right)}{x},x\right)$$

```
[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Rubi steps

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A] time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

fricas [A] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+bc)x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\log(bx+a)^3\log(x)^2 - \int \frac{2\,(bx+a)\log(x)^4 + 6\left(bx\log(bc-ad) + a\log(bc-ad)\right)\log(x)^3 + 3\left((3\,bx+2\,a)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^3*log(x)^2 - integrate(1/2*(2*(b*x + a)*log(x)^4 + 6*(b*x*
log(b*c - a*d) + a*log(b*c - a*d))*log(x)^3 + 3*((3*b*x + 2*a)*log(x)^2 + 2
*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a)^2 + 6*(b*x*lo
g(b*c - a*d)^2 + a*log(b*c - a*d)^2)*log(x)^2 - 6*((b*x + a)*log(x)^3 + 2*(
b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^2 + (b*x*log(b*c - a*d)^2 + a
*log(b*c - a*d)^2)*log(x))*log(b*x + a) + 2*(b*x*log(b*c - a*d)^3 + a*log(b
*c - a*d)^3)*log(x))/(b*x^2 + a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^3\,\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3a\int\frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)^2}{ax+bx^2}\,dx}{2}+\frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

```
[Out] 3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(a
*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**3/2
```

3.63 $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)\log^2\left(\frac{a+bx}{x(bc-ad)}\right)}{x},x\right)$$

[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]

[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]

Rubi steps

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A] time = 5.09, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]

[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+bc)x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\log(bx+a)^2\log(x)^2 - \int -\frac{(bx+a)\log(x)^3 + 2\left(bx\log(bc-ad) + a\log(bc-ad)\right)\log(x)^2 - \left((3\,bx+2\,a)\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2*log(x)^2 - integrate(-((b*x + a)*log(x)^3 + 2*(b*x*log(b
*c - a*d) + a*log(b*c - a*d))*log(x)^2 - ((3*b*x + 2*a)*log(x)^2 + 2*(b*x*l
og(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a) + (b*x*log(b*c - a*d
)^2 + a*log(b*c - a*d)^2)*log(x))/(b*x^2 + a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2\,\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}{ax+bx^2}\,dx+\frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)
```

```
[Out] a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x +
b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2
```

3.64 $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)+\text{Li}_3\left(-\frac{a}{bx}\right)+\log(x)\text{Li}_2\left(-\frac{a}{bx}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

```
[Out] -1/2*ln(1+a/b/x)*ln(x)^2+1/2*ln(b/(-a*d+b*c)+a/(-a*d+b*c)/x)*ln(x)^2+ln(x)*
polylog(2,-a/b/x)+polylog(3,-a/b/x)
```

Rubi [A] time = 0.18, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2380, 2375, 2337, 2374, 6589}

$$\text{PolyLog}\left(3,-\frac{a}{bx}\right)+\log(x)\text{PolyLog}\left(2,-\frac{a}{bx}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] -(Log[1 + a/(b*x)]*Log[x]^2)/2 + (Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*Lo
g[x]^2)/2 + Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x))]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2380

```
Int[Log[(d_.)*(u_)^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((g_.
)*(x_))^(q_.), x_Symbol] :> Int[(g*x)^q*Log[d*ExpandToSum[u, x]^r]*(a + b*L
og[c*x^n])^p, x] /; FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x
] &&  !BinomialMatchQ[u, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log(x)}{x}\,dx$$

$$= \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \frac{a\int \frac{\log^2(x)}{\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)x^2}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \int \frac{\log\left(1+\frac{a}{bx}\right)\mathrm{l}}{x}$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\mathrm{Li}_2\left(-\frac{a}{bx}\right)$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\mathrm{Li}_2\left(-\frac{a}{bx}\right)$$

Mathematica [A] time = 5.03, size = 66, normalized size = 0.80

$$\frac{1}{6}\log^2(x)\left(3\log\left(\frac{a+bx}{bcx-adx}\right)-3\log\left(\frac{bx}{a}+1\right)+\log(x)\right)+\mathrm{Li}_3\left(-\frac{bx}{a}\right)-\log(x)\mathrm{Li}_2\left(-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] (Log[x]^2*(Log[x] - 3*Log[1 + (b*x)/a] + 3*Log[(a + b*x)/(b*c*x - a*d*x)]))
/6 - Log[x]*PolyLog[2, -((b*x)/a)] + PolyLog[3, -((b*x)/a)]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

maple [C] time = 0.18, size = 450, normalized size = 5.49

$$-\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{ad-bc}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{(ad-bc)x}\right)\ln(x)^2}{4}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{(ad-bc)x}\right)^2\ln(x)^2}{4}-\frac{i\pi\,\mathrm{csgn}\left(i\left(bx+a\right)\right)\mathrm{csgr}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

```
[Out] 1/2*ln(x)^2*ln(b*x+a)-1/3*ln(x)^3+1/4*I*ln(x)^2*Pi*csgn(I/x)*csgn(I/x*(b*x+
a)/(a*d-b*c))^2+1/2*I*ln(x)^2*Pi-1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a)/(a*d-b*c))
^3-1/4*I*ln(x)^2*Pi*csgn(I/x)*csgn(I*(b*x+a)/(a*d-b*c))*csgn(I/x*(b*x+a)/(a
*d-b*c))+1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a))*csgn(I*(b*x+a)/(a*d-b*c))^2+1/4*I
*ln(x)^2*Pi*csgn(I/x*(b*x+a)/(a*d-b*c))^3+1/4*I*ln(x)^2*Pi*csgn(I/(a*d-b*c)
)*csgn(I*(b*x+a)/(a*d-b*c))^2-1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a))*csgn(I/(a*d-
b*c))*csgn(I*(b*x+a)/(a*d-b*c))-1/2*I*ln(x)^2*Pi*csgn(I/x*(b*x+a)/(a*d-b*c)
)^2+1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a)/(a*d-b*c))*csgn(I/x*(b*x+a)/(a*d-b*c))^
2-1/2*ln(x)^2*ln(a*d-b*c)-1/2*ln(x)^2*ln(1+b*x/a)-ln(x)*polylog(2,-b*x/a)+p
olylog(3,-b*x/a)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: TypeError >> unable to make sense of Maxima expression 'l
i[2]' in Sage
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)\,\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int \frac{\log{(x)}^2}{ax+bx^2}\,dx}{2}+\frac{\log{(x)}^2\log{\left(\frac{a+bx}{x(-ad+bc)}\right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

```
[Out] a*Integral(log(x)**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*
d + b*c)))/2
```

3.65 $\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)}{x\log\left(\frac{a+bx}{x(bc-ad)}\right)},x\right)$$

```
[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Rubi steps

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Mathematica [A] time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="fricas")
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="giac")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{x\ln\left(\frac{bx+a}{(-ad+bc)x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="maxima")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{x\ \ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))),x)
```

```
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{(x)}}{x\log{\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

```
[Out] Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)
```

3.66 $\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)}{x \log^2\left(\frac{a+bx}{x(bc-ad)}\right)}, x\right)$$

```
[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Rubi steps

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Mathematica [A] time = 33.66, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

fricas [A] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int\frac{\ln(x)}{x\ln\left(\frac{bx+a}{(-ad+bc)x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

```
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(bx+a)\log(x)}{a\log(bc-ad)-a\log(bx+a)+a\log(x)}-\int-\frac{bx\log(x)+bx+a}{ax\log(bc-ad)-ax\log(bx+a)+ax\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a)*log(x)/(a*log(b*c - a*d) - a*log(b*x + a) + a*log(x)) - integrat
e(-(b*x*log(x) + b*x + a)/(a*x*log(b*c - a*d) - a*x*log(b*x + a) + a*x*log(
x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{x\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2),x)
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\log(x)+bx\log(x)}{a\log\left(\frac{a+bx}{x(-ad+bc)}\right)}-\frac{\int\frac{b}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{a}{x\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{b\log(x)}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
[Out] (a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/l
og(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a*d*
x + b*c*x) + b*x/(-a*d*x + b*c*x)), x))/a
```

3.67 $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=620

$$\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_4\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

```
[Out] 1/4*m*ln(e*((b*x+a)/(d*x+c))^n)^4*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/4
*ln(e*((b*x+a)/(d*x+c))^n)^4*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/4*m*ln(e*((b*x+
a)/(d*x+c))^n)^4*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,d*(b*x+a)/b/(d*x+c))
/(-a*d+b*c)+3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,(-c*g+d*f)*(b*x+a)/
(-a*g+b*f)/(d*x+c))/(-a*d+b*c)+6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4,
d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4
,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)-6*m*n^3*polylog(5,d*(b*x
+a)/b/(d*x+c))/(-a*d+b*c)+6*m*n^3*polylog(5,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(
d*x+c))/(-a*d+b*c)
```

Rubi [A] time = 1.13, antiderivative size = 649, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$-\frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{3mn\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[(b*c - a*d)/(b*(c + d*x))])/(4*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(4*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^4*Log[h*(f + g*x)^m])/(4*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]^3*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]^3*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*
g)*(c + d*x))])/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyL
og[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)
```

```
/(c + d*x))^n]^2*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x))])/(b*c - a*d) + (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, 1 -
 (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (6*m*n^2*Log[e*((a + b*x)/(c + d
*x))^n]*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])/(b
*c - a*d) - (6*m*n^3*PolyLog[5, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d)
 + (6*m*n^3*PolyLog[5, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]
)/(b*c - a*d)
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
```

```
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{4(bc-ad)n}$$

$$= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(dm)\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}dx}{4(bc-ad)n} + \frac{((df-\ldots}{}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right.}{4(bc-ad)n}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right.}{4(bc-ad)n}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right.}{4(bc-ad)n}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right.}{4(bc-ad)n}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right.}{4(bc-ad)n}$$

Mathematica [B] time = 22.15, size = 18164, normalized size = 29.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/((b*x + a)*(d
*x + c)), x)
```

maple [F] time = 12.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}\ln\left(h\left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] -1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3*log(e)
 + 6*n*log(b*x + a)^2*log(e)^2 - 4*(n^3*log(b*x + a) - n^2*log(e))*log(d*x
+ c)^3 - 4*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^3 + 4*(log(b*x +
a) - log(d*x + c))*log((d*x + c)^n)^3 - 4*log(b*x + a)*log(e)^3 + 6*(n^3*lo
```

```
g(b*x + a)^2 - 2*n^2*log(b*x + a)*log(e) + n*log(e)^2)*log(d*x + c)^2 + 6*(
n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x +
 c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^2 + n*l
og(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*(log(b*x + a)
- log(d*x + c))*log((b*x + a)^n) - 2*log(b*x + a)*log(e))*log((d*x + c)^n)^
2 - 4*(n^3*log(b*x + a)^3 - 3*n^2*log(b*x + a)^2*log(e) + 3*n*log(b*x + a)*
log(e)^2 - log(e)^3)*log(d*x + c) - 4*(n^2*log(b*x + a)^3 - n^2*log(d*x + c
)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log(e))*log(d*x +
 c)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*
log(e) + log(e)^2)*log(d*x + c))*log((b*x + a)^n) + 4*(n^2*log(b*x + a)^3 -
 n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*l
og(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^2
+ 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log(e)
 + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*l
og(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n
))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) + integrate(1/4*(4*b*c*f*
log(e)^3*log(h) - 4*a*d*f*log(e)^3*log(h) + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3
+ (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^4 + (b*d*g*m*n^3*x^2 + a*c*g*
m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(d*x + c)^4 - 4*(b*d*g*m*n^2*x^2*
log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*
log(b*x + a)^3 + 4*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*
n^2*log(e) + a*d*g*m*n^2*log(e))*x - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*
g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a))*log(d*x + c)^3 + 4*(b*c*f*log(h) -
a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^3 - 4*(b*c*f*log(h) - a*d*f*log(h)
 + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*lo
g(d*x + c))*log((d*x + c)^n)^3 + 6*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(
e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a)^2 + 6*(b*d
*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(e)^2 + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 +
 (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^2 + (b*c*g*m*n*log(e)^2 + a*d*
g*m*n*log(e)^2)*x - 2*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g
*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a))*log(d*x + c)^2 + 6*(2*
b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n +
(b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b
*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*lo
g(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +
 a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
 (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*
m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n)^2 + 6*(2*b
*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (
b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*
c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log
(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +
```

a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
(b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m
*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log
(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m +
a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)
*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(b*c*g*log(e)^3*log
(h) - a*d*g*log(e)^3*log(h))*x - 4*(b*d*g*m*x^2*log(e)^3 + a*c*g*m*log(e)^3
+ (b*c*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x)*log(b*x + a) + 4*(b*d*g*m*x^2*l
og(e)^3 + a*c*g*m*log(e)^3 - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m*n^3
+ a*d*g*m*n^3)*x)*log(b*x + a)^3 + 3*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*
log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a)^2 + (b*c
*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x - 3*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n
*log(e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a))*log(
d*x + c) + 4*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*
n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*
d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3
+ 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m
*n*log(e))*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) +
(b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 +
(b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*g*log
(e)^2*log(h) - a*d*g*log(e)^2*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m
*log(e)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*
g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c
*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log
(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) +
a*d*g*m*n*log(e))*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) - 4*(3*b
*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 + a*c*g*m
*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2*x^2 + a
*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d*g*m*n*x
^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log
(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e
) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a
*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d*f*log(h
) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a
*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*l
og(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2
*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2
+ a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*lo
g(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*lo
g(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*l
og(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x
+ a))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b
*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d
*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c
*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m

```
*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^
2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*
log((b*x + a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a
*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^
2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**3*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=496

$$\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}+\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

```
[Out] 1/3*m*ln(e*((b*x+a)/(d*x+c))^n)^3*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/3
*ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/3*m*ln(e*((b*x+
a)/(d*x+c))^n)^3*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,d*(b*x+a)/b/(d*x+c))/(
-a*d+b*c)+2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*
g+b*f)/(d*x+c))/(-a*d+b*c)+2*m*n^2*polylog(4,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c
)-2*m*n^2*polylog(4,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [**A**] time = 0.82, antiderivative size = 517, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[(b*c - a*d)/(b*(c + d*x))])/(3*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(3*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^3*Log[h*(f + g*x)^m])/(3*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*
g)*(c + d*x))])/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog
[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(
c + d*x))^n]*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))
])/(b*c - a*d) + (2*m*n^2*PolyLog[4, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c -
 a*d) - (2*m*n^2*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x))])/(b*c - a*d)
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
```

```
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{3(bc-ad)n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(dm)\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{3(bc-ad)n} + \frac{((df-\cdots}{}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\cdots\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\cdots\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\cdots\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\cdots\right.}{3(bc-ad)n}$$

Mathematica [B] time = 10.21, size = 9211, normalized size = 18.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/((b*x + a)*(d
*x + c)), x)
```

maple [F] time = 9.23, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\ln\left(h\left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) +
3*(n^2*log(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x
+ c))*log((b*x + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)
^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log
(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(
n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a
```

```
)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))
*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*log(b*
x + a)*log(e))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) - integrate(-
1/3*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 +
 a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2
*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d
*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e)
)*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m
*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m
*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d
*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c
*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*f*log(h) - a*d*f*log(h) +
(b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d
*x + c))*log((d*x + c)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2*log
(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2 + a
*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)
^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*
x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e
) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x + a
))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g
*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m
*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*l
og(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log
(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*lo
g(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^
2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log(
(b*x + a)^n) - 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*
x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(
e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e)
 + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e
) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 +
 a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c
*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 +
 a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n))/(a*
b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g
- (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*
x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.69 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=371

$$\frac{\log\left(h(f+gx)^m\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{2n(bc-ad)}$$

```
[Out] 1/2*m*ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/2
*ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/2*m*ln(e*((b*x+
a)/(d*x+c))^n)^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln(e
*((b*x+a)/(d*x+c))^n)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*
d+b*c)-m*n*polylog(3,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)+m*n*polylog(3,(-c*g+d*
f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [A] time = 0.56, antiderivative size = 384, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$-\frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{mn\text{PolyLog}\left(3,1-\right.}{bc-a}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x
)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^2*Log[h*(f + g*x)^m])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*((a
 + b*x)/(c + d*x))^n]*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(
c + d*x))])/(b*c - a*d) - (m*n*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(
b*c - a*d) + (m*n*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c +
d*x))])/(b*c - a*d)
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
```

```
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
```

```
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\int\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{2(bc-ad)n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(dm)\int\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{2(bc-ad)n} + \frac{((df-c$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n}$$

Mathematica [B] time = 3.17, size = 1408, normalized size = 3.80

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c
 + d*x)),x]
```

```
[Out] (m*n*Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - m*n*Log[a/
b + x]^2*Log[f + g*x] - m*n*Log[c/d + x]^2*Log[f + g*x] + 2*m*n*Log[a/b + x
]*Log[a + b*x]*Log[f + g*x] - 2*m*n*Log[c/d + x]*Log[a + b*x]*Log[f + g*x]
```

```
+ 2*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[f + g*x] + 2*m*L
og[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[c/d + x]*
Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[a + b*x]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[(a + b*x)/(c + d*x
)]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x]
+ m*n*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[c +
d*x]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[c + d*x]*Log[f + g*x] + 2*m*Log[
e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 2*m*n*Log[a/b + x]*L
og[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] - 2*m*Log[a/b + x]*Log[e*((a + b
*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*n*Log[a/b + x]*Log[(
a + b*x)/(c + d*x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*Log[c/d + x]*Log[e
*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*m*n*Log[c/d +
x]*Log[(a + b*x)/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c*g)] - m*n*Log[(a + b
*x)/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] + n*L
og[a/b + x]^2*Log[h*(f + g*x)^m] + n*Log[c/d + x]^2*Log[h*(f + g*x)^m] - 2*
n*Log[a/b + x]*Log[a + b*x]*Log[h*(f + g*x)^m] + 2*n*Log[c/d + x]*Log[a + b
*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]
*Log[h*(f + g*x)^m] + 2*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(
f + g*x)^m] + 2*n*Log[a/b + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/
d + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*Log[e*((a + b*x)/(c + d*x))^n]*L
og[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[a/b + x]*Log[(b*(c + d*x))/(b*c -
a*d)]*Log[h*(f + g*x)^m] + 2*n*(m*Log[f + g*x] - Log[h*(f + g*x)^m])*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*m*(Log[e*((a + b*x)/(c + d*x))^n] -
n*Log[(a + b*x)/(c + d*x)])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*m*
n*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*m*n*
Log[(a + b*x)/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
 + d*x))] + 2*m*n*Log[f + g*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*n*
Log[h*(f + g*x)^m]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*m*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*Log[(a
+ b*x)/(c + d*x)]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))] + 2*m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(2*b*c - 2*a*d)
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c +
 (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x
 + c)), x)
```

maple [F] time = 6.68, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(h\left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(n\log\left(bx+a\right)^{2}+n\log\left(dx+c\right)^{2}-2\left(n\log\left(bx+a\right)-\log(e)\right)\log\left(dx+c\right)-2\left(\log\left(bx+a\right)-\log\left(dx+c\right)\right)\log}{2\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="maxima")
```

```
[Out] -1/2*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log
(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) + 2*(log(b*x +
 a) - log(d*x + c))*log((d*x + c)^n) - 2*log(b*x + a)*log(e))*log((g*x + f)
^m)/(b*c - a*d) + integrate(1/2*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log
(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^
2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2
+ 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) +
```

```
 a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*
d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (
b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*
x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x -
 (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^
2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n) - 2*(b*
c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2
+ a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m +
(b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d
*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)
*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.70 $\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=108

$$\frac{b\text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}$$

```
[Out] b*Unintegrable(ln(h*(g*x+f)^m)/(b*x+a)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b
*c)-d*Unintegrable(ln(h*(g*x+f)^m)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a
*d+b*c)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

```
[Out] (b*Defer[Int][Log[h*(f + g*x)^m]/((a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
, x])/(b*c - a*d) - (d*Defer[Int][Log[h*(f + g*x)^m]/((c + d*x)*Log[e*((a +
 b*x)/(c + d*x))^n]), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{b\log\left(h(f+gx)^m\right)}{(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d\log\left(h(f+gx)^m\right)}{(bc-ad)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\
&= \frac{b\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(h(f+gx)^m\right)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^m h\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

maple [A] time = 52.40, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
,x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
[Out] Timed out
```

3.71 $\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=93

$$\frac{gm\text{Int}\left(\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{n(bc-ad)}-\frac{\log\left(h(f+gx)^m\right)}{n(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

```
[Out] -ln(h*(g*x+f)^m)/(-a*d+b*c)/n/ln(e*((b*x+a)/(d*x+c))^n)+g*m*Unintegrable(1/
(g*x+f)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b*c)/n
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]^
2),x]
```

```
[Out] -(Log[h*(f + g*x)^m]/((b*c - a*d)*n*Log[e*((a + b*x)/(c + d*x))^n])) + (g*m
*Defer[Int][1/((f + g*x)*Log[e*((a + b*x)/(c + d*x))^n]), x])/((b*c - a*d)*
n)
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{(gm)\int\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{(bc-ad)n}$$

Mathematica [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2), x]
```

fricas [A] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^m h\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)^2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{(bx+a)\,(dx+c)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$gm\int \frac{1}{bcfn\log(e) - adfn\log(e) + \left(bcgn\log(e) - adgn\log(e)\right)x + \left(bcfn - adfn + \left(bcgn - adgn\right)x\right)\log\left((bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="maxima")
```

```
[Out] g*m*integrate(1/(b*c*f*n*log(e) - a*d*f*n*log(e) + (b*c*g*n*log(e) - a*d*g*
n*log(e))*x + (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((b*x + a)^n)
- (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((d*x + c)^n)), x) - (log(
(g*x + f)^m) + log(h))/(b*c*n*log(e) - a*d*n*log(e) + (b*c*n - a*d*n)*log((
b*x + a)^n) - (b*c*n - a*d*n)*log((d*x + c)^n))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{2}\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n)**2,x)
[Out] Timed out
```

3.72 $\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=114

$$\frac{b\mathrm{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\mathrm{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [A] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx &= \int \left(\frac{b\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx\\
&= \frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

fricas [A] time = 1.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(\frac{bx+a}{dx+c}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{-bx-a}{dx+c}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(-(b-d)x-a+c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}-\int -\frac{1}{\left(\left(bd-d^2\right)x^2+ac-c^2+(bc+ad-2\,cd)x\right)\log\left(bx+a\right)-\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c)) - integrate(-1/(((b*d - d^2)*x^2 + a*c - c^2 + (b*c +
a*d - 2*c*d)*x)*log(b*x + a) - ((b*d - d^2)*x^2 + a*c - c^2 + (b*c + a*d -
2*c*d)*x)*log(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{{\ln\left(\frac{a+b\,x}{c+d\,x}\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
```

```
[Out] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+bx-c-dx)\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}\,dx+\frac{\log\left(\frac{-a-bx}{c+dx}+1\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Integral(1/((c + d*x)*(a + b*x - c - d*x)*log(a/(c + d*x) + b*x/(c + d*x)))
, x) + log((-a - b*x)/(c + d*x) + 1)/(a*d*log((a + b*x)/(c + d*x)) - b*c*lo
g((a + b*x)/(c + d*x)))
```

3.73 $\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=114

$$\frac{b\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \left(\frac{b\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$=\frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

fricas [A] time = 1.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(\frac{bx+a}{dx+c}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(((b - d)*x + a - c)/(b*x + a))/((b*d*x^2 + a*c + (b*c + a*d)*x
)*log((b*x + a)/(d*x + c))^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

maple [A] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{-dx-c}{bx+a}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left((b-d)x+a-c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}-\int-\frac{1}{\left(\left(b^2-bd\right)x^2+a^2-ac+\left(a(2\,b-d)-bc\right)x\right)\log\left(bx+a\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c)) - integrate(-1/(((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b
- d) - b*c)*x)*log(b*x + a) - ((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b - d) -
 b*c)*x)*log(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{{\ln\left(\frac{a+b\,x}{c+d\,x}\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
```

```
[Out] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,(a+bx-c-dx)\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}\,dx+\frac{\log\left(1+\frac{-c-dx}{a+bx}\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*x)*(a + b*x - c - d*x)*log(a/(c + d*x) + b*x/(c + d*x)))
, x) + log(1 + (-c - d*x)/(a + b*x))/(a*d*log((a + b*x)/(c + d*x)) - b*c*lo
g((a + b*x)/(c + d*x)))
```

3.74 $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-b*x-a)/(d*x+c))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 -
(a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]), x]
 + (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c +
 d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c
 + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx = \int \frac{1}{(c+dx)(-a+c+(-b+d)}$$

$$= \int \frac{1}{(c+dx)(-a+c+(-b+d)}$$

$$= \frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad} - \frac{d\int}{}$$

Mathematica [A] time = 0.32, size = 44, normalized size = 0.98

$$\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(ad-bc)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Lo
g[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2)
,x]
```

```
[Out] Log[1 - (a + b*x)/(c + d*x)]/((-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)])
```

fricas [A] time = 0.54, size = 50, normalized size = 1.11

$$-\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(-((b - d)*x + a - c)/(d*x + c))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{((b-d)x+a-c)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)} + \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(d*x + c)*log((b*x + a)/(d*x + c))) + log
(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

maple **[C]** time = 1.16, size = 662, normalized size = 14.71

$$\frac{2i\ln(bx-dx+a-c)}{(ad-bc)\left(\pi\,\mathrm{csgn}(i(bx+a))\,\mathrm{csgn}\left(\frac{i}{dx+c}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{dx+c}\right)-\pi\,\mathrm{csgn}(i(bx+a))\,\mathrm{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2-\pi\,\mathrm{csgn}\left(\frac{i}{dx+c}\right)\mathrm{csgn}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+1/(b*x+a)/(d*x+c)/ln((b*x
+a)/(d*x+c))^2*ln(1+(-b*x-a)/(d*x+c)),x)
```

```
[Out] 2*I/(a*d-b*c)/(Pi*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))-P
i*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))-Pi*csgn(I*(b*x+a)/(d*x+c))^2*cs
gn(I/(d*x+c))+Pi*csgn(I*(b*x+a)/(d*x+c))^3+2*I*ln(b*x+a)-2*I*ln(d*x+c))*ln(
b*x-d*x+a-c)-(-I*Pi*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))
+I*Pi*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))+I*Pi*csgn(I*(b*x+a)/(d*x+c)
)^2*csgn(I/(d*x+c))+I*Pi*csgn(I/(d*x+c))*csgn(I*(b*x-d*x+a-c))*csgn(I/(d*x+
c)*(b*x-d*x+a-c))-I*Pi*csgn(I/(d*x+c))*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*Pi
*csgn(I*(b*x+a)/(d*x+c))^3+2*I*Pi*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*Pi*csgn
(I*(b*x-d*x+a-c))*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*Pi*csgn(I/(d*x+c)*(b*x-
d*x+a-c))^3-2*I*Pi+2*ln(b*x+a))/(a*d-b*c)/(-I*Pi*csgn(I*(b*x+a)/(d*x+c))^3+
I*Pi*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))+I*Pi*csgn(I*(b*x+a)/(d*x+c))
^2*csgn(I/(d*x+c))-I*Pi*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x
+c))+2*ln(b*x+a)-2*ln(d*x+c))
```

maxima **[A]** time = 1.02, size = 59, normalized size = 1.31

$$-\frac{\log(-(b-d)x-a+c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c))
```

mupad [B] time = 0.70, size = 44, normalized size = 0.98

$$\frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(c + d*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

sympy [A] time = 2.17, size = 44, normalized size = 0.98

$$\frac{\log\left(\frac{-a-bx}{c+dx}+1\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+
c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] log((-a - b*x)/(c + d*x) + 1)/(a*d*log((a + b*x)/(c + d*x)) - b*c*log((a +
b*x)/(c + d*x)))
```

3.75 $\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F] time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + Log[1 -
 (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] -Defer[Int][1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)]), x]
+ (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c +
d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c
+ d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\int \frac{1}{(a+bx)(a-c+(b-d)x)}$$

$$= -\int \frac{1}{(a+bx)(a-c+(b-d)x)}$$

$$= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+}}{}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 1.00

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + L
og[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2
),x]
```

```
[Out] -(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

fricas [A] time = 1.47, size = 49, normalized size = 1.09

$$-\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(((b - d)*x + a - c)/(b*x + a))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{((b-d)x+a-c)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)} + \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(b*x + a)*log((b*x + a)/(d*x + c))) + log
(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

maple [C] time = 1.06, size = 503, normalized size = 11.18

$$\frac{2i\ln(bx-dx+a-c)}{(ad-bc)\left(\pi\,\mathrm{csgn}\left(i\,(bx+a)\right)\mathrm{csgn}\left(\frac{i}{dx+c}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{dx+c}\right)-\pi\,\mathrm{csgn}\left(i\,(bx+a)\right)\mathrm{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2-\pi\,\mathrm{csgn}\left(\frac{i}{dx+c}\right)\mathrm{csgn}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+1/(b*x+a)/(d*x+c)/ln((b*x+
a)/(d*x+c))^2*ln(1+(-d*x-c)/(b*x+a)),x)
```

```
[Out] 2*I/(a*d-b*c)/(Pi*csgn(I*(b*x+a))*csgn(I/(d*x+c))*csgn(I*(b*x+a)/(d*x+c))-P
i*csgn(I*(b*x+a))*csgn(I*(b*x+a)/(d*x+c))^2-Pi*csgn(I/(d*x+c))*csgn(I*(b*x+
a)/(d*x+c))^2+Pi*csgn(I*(b*x+a)/(d*x+c))^3+2*I*ln(b*x+a)-2*I*ln(d*x+c))*ln(
b*x-d*x+a-c)-(I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*
x-d*x+a-c))-I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2-I*Pi
*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2+I*Pi*csgn(I/(b*x+a)*(b*x-d
*x+a-c))^3+2*ln(b*x+a))/(a*d-b*c)/(-I*Pi*csgn(I*(b*x+a))*csgn(I/(d*x+c))*cs
gn(I*(b*x+a)/(d*x+c))+I*Pi*csgn(I*(b*x+a))*csgn(I*(b*x+a)/(d*x+c))^2+I*Pi*c
sgn(I/(d*x+c))*csgn(I*(b*x+a)/(d*x+c))^2-I*Pi*csgn(I*(b*x+a)/(d*x+c))^3+2*l
n(b*x+a)-2*ln(d*x+c))
```

maxima [A] time = 1.31, size = 58, normalized size = 1.29

$$-\frac{\log((b-d)x+a-c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c))
```

mupad [B] time = 0.54, size = 44, normalized size = 0.98

$$\frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(a + b*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

sympy [A] time = 2.15, size = 44, normalized size = 0.98

$$\frac{\log\left(1+\frac{-c-dx}{a+bx}\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a
))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] log(1 + (-c - d*x)/(a + b*x))/(a*d*log((a + b*x)/(c + d*x)) - b*c*log((a +
b*x)/(c + d*x)))
```

3.76 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$

Optimal. Leaf size=560

$$\frac{a^2 n \log(a+bx)}{2b^2 g} + \frac{f \log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g^2} + \frac{x^2\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\,\mathrm{l}\right.}{2}$$

```
[Out] -1/2*a*n*x/b/g+1/2*c*n*x/d/g+1/2*a^2*n*ln(b*x+a)/b^2/g-1/2*n*x^2*ln(b*x+a)/
g-1/2*c^2*n*ln(d*x+c)/d^2/g+1/2*n*x^2*ln(d*x+c)/g+1/2*x^2*(n*ln(b*x+a)-ln(e
*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/g+1/2*f*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+
c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1
/2))/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/
(d*f^(1/2)+c*g^(1/2)))/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^
(1/2)-a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)
-c*g^(1/2)))/g^2-1/2*f*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))/
g^2-1/2*f*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*po
lylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g^2+1/2*f*n*polylog(2,(d*x+
c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g^2
```

Rubi [A] time = 0.73, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2513, 266, 43, 2416, 2395, 260, 2394, 2393, 2391}

$$-\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -(a*n*x)/(2*b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*x^
2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d*x
])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c
+ d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
 + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(
d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt
[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[
f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] - L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) - (
f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (
```

```
f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n
*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x^3\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.$$

$$= n\int\left(-\frac{x\log(a+bx)}{g} + \frac{fx\log(a+bx)}{g\left(f-gx^2\right)}\right)dx - n\int\left(-\frac{x\log(c+dx)}{g} + \frac{fx\log(c+dx)}{g\left(f-gx^2\right)}\right.$$

$$= -\frac{n\int x\log(a+bx)\,dx}{g} + \frac{n\int x\log(c+dx)\,dx}{g} + \frac{(fn)\int\frac{x\log(a+bx)}{f-gx^2}\,dx}{g} - \frac{(fn)\int\frac{x\log(c+}{f-gx^2}}{g}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{2g}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{2g}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+}{2g}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+}{2g}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+}{2g}$$

Mathematica [A] time = 0.39, size = 461, normalized size = 0.82

$$\frac{\frac{gn\left(a^2d^2\log(a+bx)-b\left(dx(ad-bc)+bc^2\log(c+dx)\right)\right)}{b^2d^2} - f\log\left(\sqrt{f}-\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - f\log\left(\sqrt{f}+\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] (-(g*x^2*Log[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*Log[a + b*x] - b*(
d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^2) - f*Log[e*((a + b*x)/(
c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - f*Log[e*((a + b*x)/(c + d*x))^n]*Lo
```

```
g[Sqrt[f] + Sqrt[g]*x] + f*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt
[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - Po
lyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) + f*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{-g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \ln\left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
[Out] Timed out
```

3.77 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=550

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{g^{3/2}}+\frac{x\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\right.}{g}$$

```
[Out] -n*(b*x+a)*ln(b*x+a)/b/g+n*(d*x+c)*ln(d*x+c)/d/g+x*(n*ln(b*x+a)-ln(e*((b*x+
a)/(d*x+c))^n)-n*ln(d*x+c))/g-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*
((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*f^(1/2)/g^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(
1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(d*x+c)*ln(d
*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(b*x+a)
*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*ln(d
*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n
*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*po
lylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*polylo
g(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*polylog(2
,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A] time = 0.57, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2513, 321, 208, 2416, 2389, 2295, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{f}\,n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\,n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\,n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}\,n\text{PolyLog}\left(2,\frac{\sqrt{}}{c}\right.}{2g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -((n*(a + b*x)*Log[a + b*x])/(b*g)) + (n*(c + d*x)*Log[c + d*x])/(d*g) + (x
*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/g - (S
qrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c +
 d*x))^n] - n*Log[c + d*x]))/g^(3/2) - (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt
[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*Log[c
+ d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
+ (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt
[g])])/(2*g^(3/2)) - (Sqrt[f]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/
(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*PolyLog[2, -((Sqrt[g]*(a
```

```
 + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog[2, (Sq
rt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog
[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^(3/2)) + (Sqrt[f]
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f-gx^2}\,dx - n\int\frac{x^2\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} + n\int\left(-\frac{\log(a+bx)}{g} + \frac{f\log(a}{g\left(f-\right.}\right.$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\,\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+b\right.}{}$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\,\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+b\right.}{}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{g}$$

Mathematica [A] time = 0.25, size = 467, normalized size = 0.85

$$\frac{-\sqrt{f}\,\log\left(\sqrt{f}-\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \sqrt{f}\,\log\left(\sqrt{f}+\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{2\sqrt{g}\,(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \sqrt{f}\,n\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] ((-2*Sqrt[g]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (2*(b*c - a*d)*S
qrt[g]*n*Log[c + d*x])/(b*d) - Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[S
qrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] +
Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])] -
 Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[g]*x]
 + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - PolyLog[
2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) - Sqrt[f]*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{-g\,x^2 + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

maxima [B] time = 2.21, size = 1047, normalized size = 1.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*(c^2/((b*c*d^3 - a*d^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*lo
g(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(
d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g))*d - 2*(c^3/((b*c*d^4 -
a*d^5)*g*x + (b*c^2*d^3 - a*c*d^4)*g) + a^3*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g) + (2*b*c^3 - 3*a*c^2*d)*log(d*x + c)/((b^2*c^2*d^3
- 2*a*b*c*d^4 + a^2*d^5)*g) - x/(b*d^2*g))*b*d^2 + 2*a*(c^2/((b*c*d^3 - a*d
^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c
*d^3 + a^2*d^4)*g))*d^2 - 2*a*c*d*(c/((b*c*d^2 - a*d^3)*g*x + (b*c^2*d - a*
c*d^2)*g) + a*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g) - a*log(d*x
+ c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g)) - 2*b*d*(a^2*log(b*x + a)/((b^3*c
 - a*b^2*d)*g) - c^2*log(d*x + c)/((b*c*d^2 - a*d^3)*g) + x/(b*d*g)) + 2*b*
c*(a*log(b*x + a)/((b^2*c - a*b*d)*g) - c*log(d*x + c)/((b*c*d - a*d^2)*g))
 - (log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*s
qrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*s
qrt(f)/g^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(
b*sqrt(f) - a*sqrt(g)) + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) -
a*sqrt(g))))*sqrt(f)/g^(3/2) + (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x -
 d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))
/(d*sqrt(f) + c*sqrt(g))))*sqrt(f)/g^(3/2) - (log(sqrt(g)*x + sqrt(f))*log(
-(d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*
x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*sqrt(f)/g^(3/2))*n - 1/2*(f*log((g
*x - sqrt(f*g))/(g*x + sqrt(f*g)))/(sqrt(f*g)*g) + 2*x/g)*log(e*((b*x + a)/
(d*x + c))^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
```

```
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

3.78 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=403

$$\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}-\frac{n\text{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n\text{Li}_2\left(\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g}-\frac{n\log(}{}$$

```
[Out] 1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g-1/2*
n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))/g+1/2*n*ln(d*x+
c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))/g-1/2*n*ln(b*x+a)*ln(b*(
f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))/g+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x
*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/g-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1
/2)-a*g^(1/2)))/g-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g+
1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g+1/2*n*polylog(2,(
d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g
```

Rubi [A] time = 0.35, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2513, 260, 2416, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}+\frac{\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -(n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2
*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])
])/(2*g) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqr
t[g])])/(2*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] -
c*Sqrt[g])])/(2*g) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x])*Log[f - g*x^2])/(2*g) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(
b*Sqrt[f] - a*Sqrt[g]))])/(2*g) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt
[f] + a*Sqrt[g])])/(2*g) + (n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] -
 c*Sqrt[g]))])/(2*g) + (n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqr
t[g])])/(2*g)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + n\int\left(\frac{\log(a+b}{2\sqrt{g}\left(\sqrt{f}-\right.}\right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2\sqrt{g}} -$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log}{2g}$$

Mathematica [A] time = 0.13, size = 413, normalized size = 1.02

$$-\frac{\log\left(\sqrt{f}-\sqrt{g}x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \log\left(\sqrt{f}+\sqrt{g}x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{\sqrt{g}a+b\sqrt{f}}\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{g}x+\sqrt{f}\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(-(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sq
rt[g]*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + n*Log
[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*
Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x
] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt
[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] - n*PolyL
og[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog[2, (d*
(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f]
```

```
+ Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]
*x))/(d*Sqrt[f] - c*Sqrt[g])])/g
```

fricas [F] time = 1.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{-g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
[Out] Timed out
```

3.79 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(c\sqrt{g}+d\sqrt{f}\right)}{(c+dx)\left(a\sqrt{g}+b\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{n\text{Li}_2\left(\frac{\left(d\sqrt{f}-c\sqrt{g}\right)(a+bx)}{\left(b\sqrt{f}-a\sqrt{g}\right)(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{L}}{}$$

```
[Out] 1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b
*f^(1/2)-a*g^(1/2)))/f^(1/2)/g^(1/2)-1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*
x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)+1
/2*n*polylog(2,(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b*f^(1/2)-a*g^(1/2)))
/f^(1/2)/g^(1/2)-1/2*n*polylog(2,(b*x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f
^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 468, normalized size of antiderivative = 1.61, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2513, 2409, 2394, 2393, 2391, 208}

$$\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] -((ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[c + d*x]))/(Sqrt[f]*Sqrt[g])) - (n*Log[a + b*x]*Log[(b*(Sqrt[
f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) + (n*Log[c +
 d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*Sqrt[f]*Sq
rt[g]) + (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[
g])])/(2*Sqrt[f]*Sqrt[g]) - (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(
d*Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) + (n*PolyLog[2, -((Sqrt[g]*(a
+ b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*Sqrt[f]*Sqrt[g]) - (n*PolyLog[2, (Sqr
t[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) - (n*PolyLog[
2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*Sqrt[f]*Sqrt[g]) + (
n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[
g])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{\log(a+bx)}{f-gx^2}\,dx - n\int \frac{\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} + n\int\left(\frac{\log(a+bx}{2\sqrt{f}\,\left(\sqrt{f}-\sqrt{\right.}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}\,x}\,dx}{2\sqrt{f}} + \frac{n}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b(}{b}\right.}{2\sqrt{f}\,\sqrt{g}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b(}{b}\right.}{2\sqrt{f}\,\sqrt{g}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b(}{b}\right.}{2\sqrt{f}\,\sqrt{g}}$$

Mathematica [A] time = 0.11, size = 421, normalized size = 1.45

$$\frac{-\log\left(\sqrt{f}-\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \log\left(\sqrt{f}+\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + n\text{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{\sqrt{g}\,a+b\sqrt{f}}\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{g}\,x+\sqrt{f}\right)}{b\sqrt{f}-a\sqrt{g}}\right) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x
] - Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[(Sqrt[g
]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[-((S
qrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + Log[
e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + n*PolyLog[2, (b
*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - n*PolyLog[2, (d*(Sqrt[f]
```

```
 - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f] + Sqrt[g
]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))/(d*
Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{-g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

maxima [A] time = 1.84, size = 349, normalized size = 1.20

$$\left(\log\left(\sqrt{g}\,x-\sqrt{f}\right)\log\left(\frac{b\sqrt{g}\,x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}+1\right)-\log\left(\sqrt{g}\,x+\sqrt{f}\right)\log\left(-\frac{b\sqrt{g}\,x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}+1\right)-\log\left(\sqrt{g}\,x-\sqrt{f}\right)\log\left(\frac{d\sqrt{g}\,x-}{d\sqrt{f}+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*
sqrt(g)) + 1) - log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*
sqrt(f) - a*sqrt(g)) + 1) - log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*s
qrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + log(sqrt(g)*x + sqrt(f))*log(-(d*sqr
t(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*
sqrt(f))/(b*sqrt(f) + a*sqrt(g))) - dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt
(f) - a*sqrt(g))) - dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)
)) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*n/sqrt(f*g)
- 1/2*log(e*((b*x + a)/(d*x + c))^n)*log((g*x - sqrt(f*g))/(g*x + sqrt(f*g)
))/sqrt(f*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

3.80 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx$

Optimal. Leaf size=518

$$\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2f}-\frac{\log(x)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\right.}{f}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))*ln(-g*x^2+f)/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*
f^(1/2)+a*g^(1/2)))/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c
*g^(1/2)))/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2))
)/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/f+n*pol
ylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/c)/f-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(
b*f^(1/2)-a*g^(1/2)))/f-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2
)))/f+1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/f+1/2*n*polyl
og(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/f
```

Rubi [A] time = 0.60, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2513, 266, 36, 29, 31, 2416, 2394, 2315, 260, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}+\frac{n\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}+\frac{n\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/
f - (n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])
/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[
g])])/(2*f) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*
Sqrt[g])])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
 - c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[c + d*x])*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
```

```
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c$$

$$= n\int\left(\frac{\log(a+bx)}{fx}-\frac{gx\log(a+bx)}{f\left(-f+gx^2\right)}\right)dx - n\int\left(\frac{\log(c+dx)}{fx}-\frac{gx\log(c+dx)}{f\left(-f+gx^2\right)}\right)dx - \frac{1}{2}\Big(n$$

$$= \frac{n\int\frac{\log(a+bx)}{x}\,dx}{f} - \frac{n\int\frac{\log(c+dx)}{x}\,dx}{f} - \frac{(gn)\int\frac{x\log(a+bx)}{-f+gx^2}\,dx}{f} + \frac{(gn)\int\frac{x\log(c+dx)}{-f+gx^2}\,dx}{f} - \frac{\Big(n\log(a}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\Big(n\log(a+bx)-\log\Big(e\Big(\frac{a+bx}{c+dx}}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\Big(n\log(a+bx)-\log\Big(e\Big(\frac{a+bx}{c+dx}}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\Big(n\log(a+bx)-\log\Big(e\Big(\frac{a+bx}{c+dx}}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\Big(n\log(a+bx)-\log\Big(e\Big(\frac{a+bx}{c+dx}}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\Big(n\log(a+bx)-\log\Big(e\Big(\frac{a+bx}{c+dx}}{f}$$

Mathematica [A] time = 0.20, size = 487, normalized size = 0.94

$$-\frac{\log\left(\sqrt{f}-\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\log\left(\sqrt{f}+\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\text{Li}_2\Big(\frac{b\left(\sqrt{f}-\sqrt{g}\right.}{\sqrt{g}\,a+b\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]

[Out] -1/2*(2*n*Log[x]*Log[1 + (b*x)/a] - 2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n]
 - 2*n*Log[x]*Log[1 + (d*x)/c] - n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*S
```

```
qrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt
[f] - Sqrt[g]*x] + n*Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[S
qrt[f] - Sqrt[g]*x] - n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]
*Log[Sqrt[f] + Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sq
rt[g]*x] + n*Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f
] + Sqrt[g]*x] + 2*n*PolyLog[2, -((b*x)/a)] - 2*n*PolyLog[2, -((d*x)/c)] -
n*PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog
[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(S
qrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] +
Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/f
```

fricas [F] time = 1.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{3}-fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^3 - f*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(-g\,x^{2}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(gx^{2}-f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)}{x\,\left(f-g\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(-g*x**2+f),x)
```

```
[Out] Timed out
```

3.81 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

Optimal. Leaf size=596

$$-\frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{f^{3/2}}+\frac{-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\,\mathrm{l}}{fx}$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x+d*n*ln(d*x+c)
/c/f+n*ln(d*x+c)/f/x+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/
x-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*
x+c))*g^(1/2)/f^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a
*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1
/2)+c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b
*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2
))/(d*f^(1/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,-(b*x+a)*g^(1/2)/
(b*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f
^(1/2)+a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1
/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+
c*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A] time = 0.58, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2513, 325, 208, 2416, 2395, 36, 29, 31, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{g}\,n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}\,n\text{PolyLog}\left(2,\cdots\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (n*L
og[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) - (Sqr
t[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[c + d*x]))/f^(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g
```

```
])])/(2*f^(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
 b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f-gx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f-gx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log\right.$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2} + \frac{g\log(a+bx)}{f\left(f-gx^2\right)}\right)dx$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\right.}{f^{3/2}}$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} - \frac{\sqrt{g}}{}$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} - \frac{\sqrt{g}}{}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+d}{fx}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+d}{fx}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+d}{fx}$$

Mathematica [A] time = 0.40, size = 479, normalized size = 0.80

$$\frac{-\sqrt{g}\log\left(\sqrt{f}-\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \sqrt{g}\log\left(\sqrt{f}+\sqrt{g}\,x\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{2\sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} + \sqrt{g}\,n\left(\log\left(\sqrt{f}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] ((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x))/(b
*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*
Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
+ a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]
)]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log
[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2
, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{4}-fx^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^4 - f*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(-g\,x^{2}+f\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

maxima [B] time = 2.38, size = 969, normalized size = 1.63

$$\frac{1}{2}\left(2\,acd\left(\frac{b^2\log\left(bx+a\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)f}+\frac{d}{\left(bc^2d-acd^2\right)fx+\left(bc^3-ac^2d\right)f}-\frac{\left(2\,bcd-ad^2\right)\log\left(dx+c\right)}{\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)f}-\frac{\log(x)}{ac^2f}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*c*d*(b^2*log(b*x + a)/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*f) + d/
((b*c^2*d - a*c*d^2)*f*x + (b*c^3 - a*c^2*d)*f) - (2*b*c*d - a*d^2)*log(d*x
 + c)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f) - log(x)/(a*c^2*f)) + 2*b*d
^2*(c/((b*c*d^2 - a*d^3)*f*x + (b*c^2*d - a*c*d^2)*f) + a*log(b*x + a)/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*f) - a*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f)) - 2*b*c*d*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) -
b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d - a*d^2)*f*x
 + (b*c^2 - a*c*d)*f)) - 2*a*d^2*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d
- a*d^2)*f*x + (b*c^2 - a*c*d)*f)) - 2*b*c*(b*log(b*x + a)/((a*b*c - a^2*d)
*f) - d*log(d*x + c)/((b*c^2 - a*c*d)*f) - log(x)/(a*c*f)) + 2*b*d*(log(b*x
 + a)/((b*c - a*d)*f) - log(d*x + c)/((b*c - a*d)*f)) + (log(sqrt(g)*x - sq
rt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g)) + 1) + dilog(-
(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*sqrt(g)/f^(3/2) - (log(
sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))
 + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))))*sqrt(g)/f
^(3/2) - (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f)
 + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g
))))*sqrt(g)/f^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(d*sqrt(g)*x + d*sqrt
(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt
(f) - c*sqrt(g))))*sqrt(g)/f^(3/2))*n - 1/2*(g*log((g*x - sqrt(f*g))/(g*x +
 sqrt(f*g)))/(sqrt(f*g)*f) + 2/(f*x))*log(e*((b*x + a)/(d*x + c))^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{x^2\,\left(f-g\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(-g*x**2+f),x)
[Out] Timed out
```

3.82 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=1046

$$-\frac{n\log(a+bx)a^2}{2b^2h}+\frac{nxa}{2bh}-\frac{cnx}{2dh}+\frac{nx^2\log(a+bx)}{2h}-\frac{gn(a+bx)\log(a+bx)}{bh^2}-\frac{nx^2\log(c+dx)}{2h}+\frac{c^2n\log(c+dx)}{2d^2h}+\frac{gn(c$$

```
[Out] 1/2*a*n*x/b/h-1/2*c*n*x/d/h-1/2*a^2*n*ln(b*x+a)/b^2/h+1/2*n*x^2*ln(b*x+a)/h
-g*n*(b*x+a)*ln(b*x+a)/b/h^2+1/2*c^2*n*ln(d*x+c)/d^2/h-1/2*n*x^2*ln(d*x+c)/
h+g*n*(d*x+c)*ln(d*x+c)/d/h^2+g*x*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*
ln(d*x+c))/h^2-1/2*x^2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/
h-1/2*(-f*h+g^2)*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x
^2+g*x+f)/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(
g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2
*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1
/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g
^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)))
)*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+
2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*
f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2
)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+
g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))
))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x
+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^
(1/2))/h^3-g*(-3*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a
)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^3/(-4*f*h+g^2)^(1/2)
```

Rubi [A] time = 1.72, antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2513, 2418, 2389, 2295, 2395, 43, 2394, 2393, 2391, 701, 634, 618, 206, 628}

$$-\frac{n\log(a+bx)a^2}{2b^2h}+\frac{nxa}{2bh}-\frac{cnx}{2dh}+\frac{nx^2\log(a+bx)}{2h}-\frac{gn(a+bx)\log(a+bx)}{bh^2}-\frac{nx^2\log(c+dx)}{2h}+\frac{c^2n\log(c+dx)}{2d^2h}+\frac{gn(c$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^3\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.$$

$$= n\int\left(-\frac{g\log(a+bx)}{h^2}+\frac{x\log(a+bx)}{h}+\frac{\left(fg+\left(g^2-fh\right)x\right)\log(a+bx)}{h^2\left(f+gx+hx^2\right)}\right)dx - n\int\left(-\right.$$

$$= \frac{gx\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2} - \frac{x^2\left(n\log(a+bx)-\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{2h}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gx\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+\right.}{h^2}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(}{2d^2}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(}{2d^2}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(}{2d^2}$$

Mathematica [A] time = 1.48, size = 1240, normalized size = 1.19

$$x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)h^2 + \frac{n\left(b\left(b\log(c+dx)c^2+d(ad-bc)x\right)-a^2d^2\log(a+bx)\right)h^2}{b^2d^2} - \frac{2g(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)h}{b} + \frac{2(bc-ad)gn\log(c+dx)h}{bd} + \frac{2fg\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (h^2*x^2*Log[e*((a + b*x)/(c + d*x))^n] - (2*g*h*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n])/b + (2*(b*c - a*d)*g*h*n*Log[c + d*x])/(b*d) + (h^2*n*(-(a^
2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^
2) + (2*f*g*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x])/Sqrt[g^2 - 4*f*h] + (g^2 - f*h)*(1 - g/Sqrt[g^2 - 4*f*h])*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x])/Sqrt[g^2 - 4*f*h]
 + (g^2 - f*h)*(1 + g/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log
[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*n*((Log[(2*h*(a + b*x))/(-(b*g)
+ 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*S
qrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g
+ Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Pol
yLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*
f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(-g + Sqrt[g^2 - 4*f*h])*n*((Log
[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*
x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*
h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + (2*f*g*h*n*((Log[(2*
h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*
c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + Pol
yLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f
*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sq
rt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(g + Sqrt[g^2 - 4*f*h
])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*
(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h]
+ 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g +
Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c
*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*h^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{h \,x^{2}+g x +f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

3.83 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=831

$$\frac{n(a+bx)\log(a+bx)}{bh}-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2}-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-\right.\right.}{2ah-b\left(g+\sqrt{g^2-\right.}}\right.}{2h^2}$$

```
[Out] n*(b*x+a)*ln(b*x+a)/b/h-n*(d*x+c)*ln(d*x+c)/d/h-x*(n*ln(b*x+a)-ln(e*((b*x+a
)/(d*x+c))^n)-n*ln(d*x+c))/h+1/2*g*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n
*ln(d*x+c))*ln(h*x^2+g*x+f)/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
)/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
)/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f
*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2
))/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f
*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*
h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1
/2))/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(
-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+
g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^2/(-4*f*h
+g^2)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2513, 2418, 2389, 2295, 2394, 2393, 2391, 703, 634, 618, 206, 628}

$$\frac{n(a+bx)\log(a+bx)}{bh}-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2}-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-\right.\right.}{2ah-b\left(g+\sqrt{g^2-\right.}}\right.}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
```

```
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g - (g^2 -
 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])
)])/(2*h^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
```

```
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + n\int\left(\frac{\log(a+bx)}{h} - \frac{(f+gx)\mathrm{l}}{h\left(f+g\right.}\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{n\int \log(a+bx)\,dx}{h} - \frac{n\int \frac{(f+gx}{f+}}{}$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -\right.}{h}$$

<u>**Mathematica**</u> **[A]** time = 4.11, size = 1105, normalized size = 1.33

$$\frac{2dh\sqrt{g^2-4fh}\,(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2bdfh\log\left(g+2hx-\sqrt{g^2-4fh}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + bdg\left(g-\sqrt{g^2-4fh}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (2*d*h*Sqrt[g^2 - 4*f*h]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*(b*c
- a*d)*h*Sqrt[g^2 - 4*f*h]*n*Log[c + d*x] - 2*b*d*f*h*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + b*d*g*(g - Sqrt[g^2 - 4*f*h]
)*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + 2*b*d
*f*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - b*
d*g*(g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2
 - 4*f*h] + 2*h*x] + 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*
f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 -
 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(
-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - b
*d*g*(g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sq
rt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*
h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4
*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g
 + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - 2*b
*d*f*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(
2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f
*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(
g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]) + b*d*g*(g + Sqrt[g^2 - 4*f*h])*n*((L
og[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x
))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x]
 + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2
 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(
g + Sqrt[g^2 - 4*f*h]))]))/(2*b*d*h^2*Sqrt[g^2 - 4*f*h])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \ln \left(e \left(\frac{b x+a}{d x+c}\right)^{n}\right)}{h \,x^{2}+g x+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
[Out] Timed out
```

3.84 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=685

$$-\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}-\frac{\log\left(f+gx+hx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2h}$$

```
[Out] -1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/h+
1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)
^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h
+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h+1
/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g
^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*
(1-g/(-4*f*h+g^2)^(1/2))/h+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2
))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*
x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(
1+g/(-4*f*h+g^2)^(1/2))/h+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g
^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-
d*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-g*arctanh((2*h*x+g)/(
-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h/(-
4*f*h+g^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2513, 2418, 2394, 2393, 2391, 634, 618, 206, 628}

$$\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{2h}-\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] -((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h*Sqrt[g^2 - 4*f*h])) + ((1 - g/Sqrt[
g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a
*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log
[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^
2 - 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*
(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
```

```
h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h) - ((n*Log[a + b*
x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])
/(2*h) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b
*(g - Sqrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2,
 (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h) - ((1 - g/Sqrt
[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*
h]))])/(2*h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c
*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \right.$$

$$= n\int\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx - n\int\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4f}}\right.}{g-\sqrt{g^2}}\right.$$

$$= -\frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h} + \left(\left(1-\frac{g}{\sqrt{g^2-}}\right.\right.$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4f}}\right.}{}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4f}}\right.}{}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4f}}\right.}{}$$

Mathematica [A] time = 0.73, size = 539, normalized size = 0.79

$$\frac{\left(\sqrt{g^2-4fh}-g\right)\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \left(\sqrt{g^2-4fh}+g\right)\log\left(\sqrt{g^2-4fh}+g+2hx\right)\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] ((-g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 -
 4*f*h] + 2*h*x] + (g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*L
og[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + (g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
```

```
 + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d
*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + P
olyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-
g + Sqrt[g^2 - 4*f*h]))]) - (g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))
/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h - d*(g +
 Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(
g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - Pol
yLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f
*h]))]))/(2*h*Sqrt[g^2 - 4*f*h])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
[Out] integral(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \ln\left(e \left(\frac{a+b x}{c+d x}\right)^n\right)}{h x^2 + g x + f} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=401

$$-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)\left(c^2h-cdg+d^2f\right)}{(c+dx)\left(-\sqrt{g^2-4fh}\,(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)\left(c^2h-c\right.}{(c+dx)\left(\sqrt{g^2-4fh}\,(bc-ad)+2\right.}\right.}{\sqrt{g^2-4fh}}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*
d*f-b*c*g-a*d*g+2*a*c*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+
ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d
*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)-n
*polylog(2,2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c
*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+n*polylog(2,2*(c^2*h-
c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*
h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 545, normalized size of antiderivative = 1.36, number of steps used = 19, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2513, 2418, 2394, 2393, 2391, 618, 206}

$$\frac{n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{\sqrt{g^2-4fh}}-\frac{n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{\sqrt{g^2-4fh}}-\frac{n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{\sqrt{g^2-4fh}}+\frac{n\,\text{PolyLog}\left(\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (2*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x
)/(c + d*x))^n] - n*Log[c + d*x]))/Sqrt[g^2 - 4*f*h] + (n*Log[a + b*x]*Log[
-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))]
)/Sqrt[g^2 - 4*f*h] - (n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h
*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/Sqrt[g^2 - 4*f*h] - (n*Log[a +
b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4
*f*h])))])/Sqrt[g^2 - 4*f*h] + (n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/Sqrt[g^2 - 4*f*h] + (n
*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2
- 4*f*h] - (n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])
)])/Sqrt[g^2 - 4*f*h] - (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[
g^2 - 4*f*h]))])/Sqrt[g^2 - 4*f*h] + (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h -
 d*(g + Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2 - 4*f*h]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
```

```
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\,\mathrm{lo}\right.$$

$$= n\int\left(\frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g-\sqrt{g^2-4fh}+2hx\right)} - \frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g+\sqrt{g^2-4fh}+2hx\right)}\right)dx - n$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{(2hn)\int \frac{\log(a+}{g-\sqrt{g^2-4}}}{\sqrt{g^2-4f}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log}{\sqrt{g}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log}{\sqrt{g}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log}{\sqrt{g}}$$

Mathematica [A] time = 0.35, size = 515, normalized size = 1.28

$$\frac{\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \log\left(\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\mathrm{Li}_2\left(\frac{b\left(-g-2hx+\sqrt{g^2-}\right.}{2ah+b\left(\sqrt{g^2-4fh}\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (-(n*Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])]*Log[g - Sq
rt[g^2 - 4*f*h] + 2*h*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2
 - 4*f*h] + 2*h*x] + n*Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4
*f*h])]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + n*Log[(2*h*(a + b*x))/(2*a*h -
 b*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - n*Log[(2*h*(c +
d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + n*PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(d*g) + 2*c*h + d*
Sqrt[g^2 - 4*f*h])] - n*PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*
a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] + n*PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h]
 + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - n*PolyLog[2, (d*(g + Sqr
t[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2 -
4*f*h]
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$

Optimal. Leaf size=800

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{\right.}}{2ah-b\left(g+\sqrt{\right.}}\right.}{2f}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f+n*polylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/
c)/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h
+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/f+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-
4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2)
)/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*
f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2
))))*(1-g/(-4*f*h+g^2)^(1/2))/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*
ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)
)))*(1+g/(-4*f*h+g^2)^(1/2))/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*
f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2
*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f-g*arctanh((2*h*x
+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))
/f/(-4*f*h+g^2)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 800, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2513, 2418, 2394, 2315, 2393, 2391, 705, 29, 634, 618, 206, 628}

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{\right.}}{2ah-b\left(g+\sqrt{\right.}}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
```

```
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h]))))])/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
```

```
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f+gx+hx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.$$

$$= n\int \left(\frac{\log(a+bx)}{fx} + \frac{(-g-hx)\log(a+bx)}{f\left(f+gx+hx^2\right)}\right)dx - n\int \left(\frac{\log(c+dx)}{fx} + \frac{(-g-hx)\log(c}{f\left(f+gx+h\right.}\right.$$

$$= -\frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f} + \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} + \frac{n\int \frac{(-g}{}}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\right.\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+b\right.}{f\sqrt{g}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+b\right.}{f\sqrt{g}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+b\right.}{f\sqrt{g}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+b\right.}{f\sqrt{g}}$$

Mathematica [A] time = 0.94, size = 625, normalized size = 0.78

$$-\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt[g
^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h
*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x
)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
+ b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*
g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + Po
lyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
- 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g
 + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(
(Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d
*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g
^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d
*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^3+gx^2+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^3 + g*x^2 + f*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(h\,x^{2}+gx+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)}{x\,\left(h\,x^{2}+g\,x+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx$

Optimal. Leaf size=995

$$\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log(x)}{f^2}-\frac{bn\log(a+bx)}{af}-\frac{gn\log\left(-\frac{bx}{a}\right)\ldots}{f^2}$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x-g*n*ln(-b*x/a
)*ln(b*x+a)/f^2+d*n*ln(d*x+c)/c/f+n*ln(d*x+c)/f/x+g*n*ln(-d*x/c)*ln(d*x+c)/
f^2+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/x+g*ln(x)*(n*ln(b
*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2-1/2*g*(n*ln(b*x+a)-ln(e*((
b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f^2-g*n*polylog(2,1+b*x/a)/
f^2+g*n*polylog(2,1+d*x/c)/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h
+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h*(
b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*
h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2)
)/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h
*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/
2))/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(-
2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g
^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2/(-4*f*h+
g^2)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 995, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2513, 2418, 2395, 36, 29, 31, 2394, 2315, 2393, 2391, 709, 800, 634, 618, 206, 628}

$$\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log(x)}{f^2}-\frac{bn\log(a+bx)}{af}-\frac{gn\log\left(-\frac{bx}{a}\right)\ldots}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*
x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2
 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x)
 + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] -
Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) +
 (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x
]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^
2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sq
rt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) +
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g -
 (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4
*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[
a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h
*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[
2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*Pol
yLog[2, 1 + (d*x)/c])/f^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f+gx+hx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+}{c+}\right.\right.\right.$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2} - \frac{g\log(a+bx)}{f^2x} + \right.$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + \frac{n\int \frac{\left(g^2-fh+ghx\right)\log(a+bx)}{f+gx+hx^2}\,dx}{f^2} - \frac{n\int \frac{\left(g^2\right.}{}}{}$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+d}{f^2}$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+d}{f^2}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} +$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} +$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} +$$

Mathematica [A] time = 0.88, size = 721, normalized size = 0.72

$$\frac{\left(\frac{g^2-2fh}{\sqrt{g^2-4fh}}+g\right)\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\left(\frac{2fh-g^2}{\sqrt{g^2-4fh}}+g\right)\log\left(\sqrt{g^2-4fh}+g+2hx\right)\log\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] ((-2*f*Log[e*((a + b*x)/(c + d*x))^n])/x - 2*g*Log[x]*Log[e*((a + b*x)/(c +
 d*x))^n] + (2*f*n*((b*c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x
]))/(a*c) + (g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x
))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + (g + (-g^2 + 2*f*h)/Sqrt[g^2 - 4
*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] +
2*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)]) - ((g^2 - 2*f*h + g*Sqrt[g^2 - 4*f*h])*n*((Log[
(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x
))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h
*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*S
qrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h
 + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g^2 - 2*f*h - g*Sqr
t[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))
] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[
g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*
a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] +
2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^4+gx^3+fx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^4 + g*x^3 + f*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(h\,x^{2}+gx+f\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re details)Is 4*f*h-g^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)}{x^{2}\,\left(h\,x^{2}+g\,x+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

3.88 $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

Optimal. Leaf size=46

$$-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{b}$$

```
[Out] -ln(a/(b*x+a))*ln(c*x/(b*x+a))/b-polylog(2,1-a/(b*x+a))/b
```

Rubi [A] time = 0.16, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2488, 2411, 2343, 2333, 2315}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] -((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]/
b
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\int\frac{\log\left(\frac{a}{a+bx}\right)}{x(a+bx)}\,dx}{b}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\,\text{Subst}\left(\int\frac{\log\left(\frac{a}{x}\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx,x,a+bx\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\,\text{Subst}\left(\int\frac{\log(ax)}{\left(-\frac{a}{b}+\frac{1}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\,\text{Subst}\left(\int\frac{\log(ax)}{\frac{1}{b}-\frac{ax}{b}}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(\frac{bx}{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 84, normalized size = 1.83

$$-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(\frac{a+bx}{a}\right)}{b}+\frac{\log^2\left(\frac{a}{a+bx}\right)}{2b}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[a/(a + b*x)])/b + Log[a/(a + b*x)]^2/(2*b) - (Log[a/(a
 + b*x)]*Log[(c*x)/(a + b*x)])/b - PolyLog[2, (a + b*x)/a]/b
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(c*x/(b*x + a))/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))/(b*x + a), x)
```

maple [B] time = 0.08, size = 97, normalized size = 2.11

$$-\frac{\ln\left(-\frac{\left(-\frac{ac}{(bx+a)b}+\frac{c}{b}\right)b-c}{c}\right)\ln\left(-\frac{ac}{(bx+a)b}+\frac{c}{b}\right)}{b}-\frac{\operatorname{dilog}\left(-\frac{\left(-\frac{ac}{(bx+a)b}+\frac{c}{b}\right)b-c}{c}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))/(b*x+a),x)
```

```
[Out] -dilog(-(b*(1/b*c-a*c/b/(b*x+a))-c)/c)/b-ln(1/b*c-a*c/b/(b*x+a))*ln(-(b*(1/
b*c-a*c/b/(b*x+a))-c)/c)/b
```

maxima [B] time = 0.68, size = 95, normalized size = 2.07

$$\frac{\log\left(bx+a\right)\log\left(\frac{cx}{bx+a}\right)}{b}-\frac{\frac{c\log(bx+a)^2}{b}-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)c}{b}}{2\,c}+\frac{\left(c\log\left(bx+a\right)-c\log(x)\right)\log\left(bx+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(c*x/(b*x + a))/b - 1/2*(c*log(b*x + a)^2/b - 2*(log(b*x/a
+ 1)*log(x) + dilog(-b*x/a))*c/b)/c + (c*log(b*x + a) - c*log(x))*log(b*x +
 a)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x)/(a + b*x))/(a + b*x),x)
```

```
[Out] int(log((c*x)/(a + b*x))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(c*x/(a + b*x))/(a + b*x), x)
```

3.89 $\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$

Optimal. Leaf size=20

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[Out] 1/3*ln(c*x/(b*x+a))^3/a
```

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2505}

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]}
, Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c -
 a*d)), x] /; FreeQ[h, x]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Mathematica [A] time = 0.10, size = 20, normalized size = 1.00

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

fricas [A] time = 0.68, size = 18, normalized size = 0.90

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)^2}{(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2/((b*x + a)*x), x)
```

maple [A] time = 0.04, size = 29, normalized size = 1.45

$$\frac{\ln\left(-\frac{ac}{(bx+a)b}+\frac{c}{b}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

```
[Out] 1/3/a*ln(-1/(b*x+a)*a/b*c+1/b*c)^3
```

maxima [B] time = 0.61, size = 141, normalized size = 7.05

$$-\left(\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}\right)\log\left(\frac{cx}{bx+a}\right)^2-\frac{\left(c\log(bx+a)^2-2\,c\log(bx+a)\log(x)+c\log(x)^2\right)\log\left(\frac{cx}{bx+a}\right)}{ac}-\frac{c^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)*log(c*x/(b*x + a))^2 - (c*log(b*x + a)^2 - 2*c
*log(b*x + a)*log(x) + c*log(x)^2)*log(c*x/(b*x + a))/(a*c) - 1/3*(c^2*log(
```

```
b*x + a)^3 - 3*c^2*log(b*x + a)^2*log(x) + 3*c^2*log(b*x + a)*log(x)^2 - c^
2*log(x)^3)/(a*c^2)
```

mupad [B] time = 0.26, size = 18, normalized size = 0.90

$$\frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x)/(a + b*x))^2/(x*(a + b*x)),x)
[Out] log((c*x)/(a + b*x))^3/(3*a)
```

sympy [A] time = 0.29, size = 14, normalized size = 0.70

$$\frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
[Out] log(c*x/(a + b*x))**3/(3*a)
```

3.90 $\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\text{Li}_2\left(1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\text{Li}_3\left(1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\text{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}$$

```
[Out] -ln(c*x/(b*x+a))^2*polylog(2,1-a/(b*x+a))/a+2*ln(c*x/(b*x+a))*polylog(3,1-a
/(b*x+a))/a-2*polylog(4,1-a/(b*x+a))/a
```

Rubi [A] time = 0.17, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - (2*PolyLog[4, 1 - a/(a + b*x)])/a
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\mathrm{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+2\int \frac{\log\left(\frac{cx}{a+bx}\right)\mathrm{Li}_2\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx\\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\mathrm{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\mathrm{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-2\int \frac{\mathrm{Li}_3\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx\\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\mathrm{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\mathrm{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-\frac{2\mathrm{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 76, normalized size = 0.93

$$-\frac{\mathrm{Li}_2\left(\frac{bx}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\mathrm{Li}_3\left(\frac{bx}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\mathrm{Li}_4\left(\frac{bx}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, (b*x)/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, (b*x)/(a + b*x)])/a - (2*PolyLog[4, (b*x)/(a + b*x)])/a
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{bx^2+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas"
)
```

```
[Out] integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{a}{bx+a}\right)\ln\left(\frac{cx}{bx+a}\right)^{2}}{(bx+a)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

```
[Out] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(bx+a)^{4}-4\log(bx+a)^{3}\log(x)}{4a}+\int \frac{a\log(a)\log(c)^{2}+2a\log(a)\log(c)\log(x)+a\log(a)\log(x)^{2}+\left(a\left(\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima"
)
```

```
[Out] 1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log(
c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(c)
) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x) +
a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a^2*x
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^{2}\,\ln\left(\frac{a}{a+b\,x}\right)}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)),x)
```

```
[Out] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)^{2}}{x\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

3.91 $\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}+\frac{2\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}-\frac{2\text{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[Out] -ln(e*(b*x+a)/(d*x+c))^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+2*ln
(e*(b*x+a)/(d*x+c))*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g-2*polylog
(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.055, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}+\frac{2\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}-\frac{2\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((c + d
*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]
)/((b*c - a*d)*g)) + (2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*c -
a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*PolyLog[4, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*c - a*d)*g)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
```

```
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\int\frac{\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}$$

$$= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2\int\frac{\mathrm{Li}_3\left(1-\right.}{(a+bx)}}{g}$$

$$= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2\mathrm{Li}_4\left(1-\right.}{(bc-}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 0.73

$$\frac{-\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)+2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)-2\mathrm{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/(
(c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (-(Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
+ 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] -
2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\frac{bc-ad}{bdx+bc}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{bdgx^2+acg+(bc+ad)gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="fricas")
```

```
[Out] integral(log((b*c - a*d)/(b*d*x + b*c))*log((b*e*x + a*e)/(d*x + c))^2/(b*d
*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2 \log\left(\frac{bc-ad}{(dx+c)b}\right)}{\left(bgx+ag\right)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/((d*x + c)*b))/((b*g
*x + a*g)*(d*x + c)), x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{(bx+a)e}{dx+c}\right)^2 \ln\left(\frac{-ad+bc}{(dx+c)b}\right)}{(dx+c)\left(bgx+ag\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*d+b*c)/b/(d*x+c))*ln((b*x+a)/(d*x+c)*e)^2/(d*x+c)/(b*g*x+a*g),x)
```

```
[Out] int(ln((-a*d+b*c)/b/(d*x+c))*ln((b*x+a)/(d*x+c)*e)^2/(d*x+c)/(b*g*x+a*g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\log(bx+a)\log(dx+c)^3-\log(dx+c)^4}{4\left(bcg-adg\right)}-\int \frac{\left(\left(d\log(bc-ad)-d\log(b)\right)a-\left(c\log(bc-ad)-c\log(b)\right)b\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="maxima")
```

```
[Out] -1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - int
egrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)
```

```
*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d*log(e))*a - (c*(log(b
*c - a*d) + 2*log(e)) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x + a))
*log(d*x + c)^2 + (d*log(b*c - a*d)*log(e)^2 - d*log(b)*log(e)^2)*a - (c*lo
g(b*c - a*d)*log(e)^2 - c*log(b)*log(e)^2)*b + 2*((d*log(b*c - a*d)*log(e)
- d*log(b)*log(e))*a - (c*log(b*c - a*d)*log(e) - c*log(b)*log(e))*b)*log(b
*x + a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d)*log(e) - 2*d*lo
g(b)*log(e) + d*log(e)^2)*a - (2*c*log(b)*log(e) - (2*log(b*c - a*d)*log(e)
 + log(e)^2)*c)*b - 2*((d*log(b*c - a*d) - d*log(b) + d*log(e))*a - (c*(log
(b*c - a*d) + log(e)) - c*log(b))*b)*log(b*x + a))*log(d*x + c))/(a*b*c^2*g
 - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a^2*d^2*g)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\,\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)}{\left(a\,g+b\,g\,x\right)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
+ b*g*x)*(c + d*x)),x)
```

```
[Out] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
+ b*g*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d\int\frac{\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^3}{c+dx}\,dx}{3g\,(ad-bc)}-\frac{\log\left(\frac{-ad+bc}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adg-3bcg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))**2/(d*x+c)/(b*g*x+
a*g),x)
```

```
[Out] -d*Integral(log(a*e/(c + d*x) + b*e*x/(c + d*x))**3/(c + d*x), x)/(3*g*(a*d
 - b*c)) - log((-a*d + b*c)/(b*(c + d*x)))*log(e*(a + b*x)/(c + d*x))**3/(3
*a*d*g - 3*b*c*g)
```

3.92 $\displaystyle\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=160

$$-\frac{\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}+\frac{2n\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}-\frac{2n^2\text{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+
2*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
-2*n^2*polylog(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A] time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}+\frac{2n\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}-\frac{2n^2\text{PolyLog}\left(4,1-\frac{bc}{b(c}\right.}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/((c +
 d*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x)
)])/((b*c - a*d)*g)) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1 - (
b*c - a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[4, 1 - (b*c - a
*d)/(b*(c + d*x))])/((b*c - a*d)*g)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
```

```
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{(2n)\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}$$

$$= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} -$$

$$= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} -$$

Mathematica [B] time = 0.45, size = 559, normalized size = 3.49

$$\frac{\log\left(\frac{a+bx}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)\left(3\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-3n\log\left(\frac{a+bx}{c+dx}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n^2\log^2\left(\frac{a+bx}{c+dx}\right)\right)+3n\left(-2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])
/((c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c + d
*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[c +
d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c + d*x
] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(a + b*
```

```
x))/(-(b*c) + a*d)]))/2 + 3*n*(-Log[e*((a + b*x)/(c + d*x))^n] + n*Log[(a +
 b*x)/(c + d*x)])*(Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)
] + 2*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]) - n^2*(Log[(a + b*x)/(c + d*x)]^3*
Log[(b*c - a*d)/(b*c + b*d*x)] + 3*Log[(a + b*x)/(c + d*x)]^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))] - 6*Log[(a + b*x)/(c + d*x)]*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))] + 6*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]))/(3*(b*c
- a*d)*g)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\log\left(\frac{bc-ad}{bdx+bc}\right)}{bdgx^{2}+acg+(bc+ad)gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/(b*d*x + b*c))/(b
*d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\log\left(\frac{bc-ad}{(dx+c)b}\right)}{\left(bgx+ag\right)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

maple [F] time = 10.22, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\ln\left(\frac{-ad+bc}{(dx+c)b}\right)}{(dx+c)\left(bgx+ag\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/(d*x+c)/b)/(d*x+c)/(b*g*x+a*g
),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/(d*x+c)/b)/(d*x+c)/(b*g*x+a*g
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}\log\left(\frac{bc-ad}{(dx+c)b}\right)}{\left(bgx+ag\right)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="maxima")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{2}}{\left(a\,g+b\,g\,x\right)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)),x)
```

```
[Out] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x)
```

```
[Out] Timed out
```

3.93 $\int \log \left(\frac{c(b+ax)}{x}\right) dx$

Optimal. Leaf size=25

$$x \log \left(ac + \frac{bc}{x}\right) + \frac{b \log(ax + b)}{a}$$

```
[Out] x*ln(a*c+b*c/x)+b*ln(a*x+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2453, 2448, 263, 31}

$$x \log \left(ac + \frac{bc}{x}\right) + \frac{b \log(ax + b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x],x]
```

```
[Out] x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(\frac{c(b+ax)}{x}\right)dx &= \int \log\left(ac+\frac{bc}{x}\right)dx \\
&= x\log\left(ac+\frac{bc}{x}\right)+(bc)\int \frac{1}{\left(ac+\frac{bc}{x}\right)x}\,dx \\
&= x\log\left(ac+\frac{bc}{x}\right)+(bc)\int \frac{1}{bc+acx}\,dx \\
&= x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(b+ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.12

$$
\frac{(ax+b)\log\left(\frac{c(ax+b)}{x}\right)}{a}+\frac{b\log(x)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x],x]
[Out] (b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

fricas [A] time = 0.71, size = 29, normalized size = 1.16

$$
\frac{ax\log\left(\frac{acx+bc}{x}\right)+b\log\left(ax+b\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="fricas")
[Out] (a*x*log((a*c*x + b*c)/x) + b*log(a*x + b))/a
```

giac [B] time = 0.18, size = 153, normalized size = 6.12

$$
\frac{b^2c^2\left(\frac{\log\left(\frac{|acx+bc|}{|x|}\right)}{ac}-\frac{\log\left(\left|-ac+\frac{acx+bc}{x}\right|\right)}{ac}\right)-\frac{b^2c^2\log\left(-\left(b-\frac{a}{\frac{a}{b}-\frac{acx+bc}{bcx}}\right)c\left(\frac{a}{b}-\frac{acx+bc}{bcx}\right)\right)}{ac-\frac{acx+bc}{x}}}{bc}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

```
[Out] (b^2*c^2*(log(abs(a*c*x + b*c)/abs(x))/(a*c) - log(abs(-a*c + (a*c*x + b*c)
/x))/(a*c)) - b^2*c^2*log(-(b - a/(a/b - (a*c*x + b*c)/(b*c*x)))*c*(a/b - (
a*c*x + b*c)/(b*c*x)))/(a*c - (a*c*x + b*c)/x))/(b*c)
```

maple [A] time = 0.12, size = 44, normalized size = 1.76

$$x \ln\left(ac+\frac{bc}{x}\right)-\frac{b \ln\left(\frac{bc}{x}\right)}{a}+\frac{b \ln\left(ac+\frac{bc}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x),x)
```

```
[Out] -b/a*ln(b*c/x)+x*ln(a*c+b*c/x)+b*ln(a*c+b*c/x)/a
```

maxima [A] time = 0.63, size = 25, normalized size = 1.00

$$x \log\left(\frac{(ax+b)c}{x}\right)+\frac{b \log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(a*x + b)/a
```

mupad [B] time = 0.08, size = 25, normalized size = 1.00

$$x\ \ln\left(\frac{c\ (b+a\,x)}{x}\right)+\frac{b\ \ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x),x)
```

```
[Out] x*log((c*(b + a*x))/x) + (b*log(b + a*x))/a
```

sympy [A] time = 0.27, size = 20, normalized size = 0.80

$$x \log\left(\frac{c\left(ax+b\right)}{x}\right)+\frac{b \log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x),x)
```

```
[Out] x*log(c*(a*x + b)/x) + b*log(a*x + b)/a
```

3.94 $\int \log^2\left(\frac{c(b+ax)}{x}\right) dx$

Optimal. Leaf size=67

$$\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}-\frac{2b\text{Li}_2\left(\frac{b}{ax}+1\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^2/a-2*b*ln(c*(a+b/x))*ln(-b/a/x)/a-2*b*polylog(2,1+b/
a/x)/a
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2453, 2449, 2454, 2394, 2315}

$$-\frac{2b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*Log[c*(a + b/x)]*Log[-(b/(a*x))])
/a - (2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)dx = \int \log^2\left(ac+\frac{bc}{x}\right)dx$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} + \frac{(2b)\int \frac{\log\left(ac+\frac{bc}{x}\right)}{x}dx}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{(2b)\,\text{Subst}\left(\int \frac{\log(ac+bcx)}{x}dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{\left(2b^2c\right)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{ac+bcx}dx, \right.}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{2b\text{Li}_2\left(1+\frac{b}{ax}\right)}{a}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.94

$$\frac{\log\left(\frac{c(ax+b)}{x}\right)\left((ax+b)\log\left(\frac{c(ax+b)}{x}\right)-2b\log\left(-\frac{b}{ax}\right)\right)-2b\text{Li}_2\left(\frac{b}{ax}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] (Log[(c*(b + a*x))/x]*(-2*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))/x
]) - 2*b*PolyLog[2, 1 + b/(a*x)])/a
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\frac{acx+bc}{x}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{(ax+b)c}{x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^2, x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{(ax+b)\,c}{x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x)^2,x)
```

```
[Out] int(ln(c*(a*x+b)/x)^2,x)
```

maxima [A] time = 0.66, size = 113, normalized size = 1.69

$$x\log\left(\frac{(ax+b)c}{x}\right)^2 + \frac{2\,b\log(ax+b)\log\left(\frac{(ax+b)c}{x}\right)}{a} + \frac{\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\left(c\log(ax+b)-c\log(x)\right)b}{a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a*
x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*
x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\ (b+a\,x)}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x)^2,x)
```

```
[Out] int(log((c*(b + a*x))/x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2b\int \frac{\log\left(ac+\frac{bc}{x}\right)}{ax+b}\,dx + x\log\left(\frac{c\,(ax+b)}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**2,x)
```

```
[Out] 2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2
```

3.95 $\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal. Leaf size=97

$$-\frac{6b\text{Li}_2\left(\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\text{Li}_3\left(\frac{b}{ax}+1\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^3/a-3*b*ln(c*(a+b/x))^2*ln(-b/a/x)/a-6*b*ln(c*(a+b/x)
)*polylog(2,1+b/a/x)/a+6*b*polylog(3,1+b/a/x)/a
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2453, 2449, 2454, 2396, 2433, 2374, 6589}

$$-\frac{6b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\text{PolyLog}\left(3,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*Log[c*(a + b/x)]^2*Log[-(b/(a*x))
])/a - (6*b*Log[c*(a + b/x)]*PolyLog[2, 1 + b/(a*x)])/a + (6*b*PolyLog[3, 1
 + b/(a*x)])/a
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)dx = \int \log^3\left(ac+\frac{bc}{x}\right)dx$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} + \frac{(3b)\int \frac{\log^2\left(ac+\frac{bc}{x}\right)}{x}dx}{a}$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{(3b)\,\text{Subst}\left(\int \frac{\log^2(ac+bcx)}{x}dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b^2c)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log(ac}{ac+bcx}\right.}{a}$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b)\,\text{Subst}\left(\int \frac{\log(x)\log\left(-\frac{b\left(-\frac{a}{b}+\right.}{a}\right.}{x}\right.}{a}$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right.}{a}$$

$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right.}{a}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.94

$$\frac{-6b\text{Li}_2\left(\frac{b}{ax}+1\right)\log\left(\frac{c(ax+b)}{x}\right)+\left((ax+b)\log\left(\frac{c(ax+b)}{x}\right)-3b\log\left(-\frac{b}{ax}\right)\right)\log^2\left(\frac{c(ax+b)}{x}\right)+6b\text{Li}_3\left(\frac{b}{ax}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] (Log[(c*(b + a*x))/x]^2*(-3*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))
/x]) - 6*b*Log[(c*(b + a*x))/x]*PolyLog[2, 1 + b/(a*x)] + 6*b*PolyLog[3, 1
+ b/(a*x)])/a
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\frac{acx+bc}{x}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^3, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{(ax+b)\,c}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a*x+b)*c/x)^3,x)
```

```
[Out] int(ln((a*x+b)*c/x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ax+b)\log(ax+b)^3 + 3\left(ax\log(c) - ax\log(x)\right)\log(ax+b)^2}{a} + \int \frac{ax\log(c)^3 + b\log(c)^3 - (ax+b)\log(x)^3 + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")
```

```
[Out] ((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a +
 integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c)
+ b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x + b
)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(a*x*
log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x)^3,x)
```

```
[Out] int(log((c*(b + a*x))/x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$3b\int\frac{\log\left(ac+\frac{bc}{x}\right)^2}{ax+b}\,dx+x\log\left(\frac{c\,(ax+b)}{x}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**3,x)
```

```
[Out] 3*b*Integral(log(a*c + b*c/x)**2/(a*x + b), x) + x*log(c*(a*x + b)/x)**3
```

3.96 $\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=28

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

```
[Out] 2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2486, 31}

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2],x]
```

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx &= x\log\left(\frac{c(b+ax)^2}{x^2}\right)+(2b)\int \frac{1}{b+ax}\,dx\\ &= \frac{2b\log(b+ax)}{a}+x\log\left(\frac{c(b+ax)^2}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2],x]
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

fricas [A] time = 0.71, size = 42, normalized size = 1.50

$$\frac{ax\log\left(\frac{a^2cx^2+2\,abcx+b^2c}{x^2}\right)+2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")
[Out] (a*x*log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*log(a*x + b))/a
```

giac [A] time = 0.18, size = 29, normalized size = 1.04

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)+\frac{2\,b\log\left(|ax+b|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(abs(a*x + b))/a
```

maple [A] time = 0.06, size = 40, normalized size = 1.43

$$x\ln\left(\left(a+\frac{b}{x}\right)^2c\right)-\frac{2b\ln\left(\frac{1}{x}\right)}{a}+\frac{2b\ln\left(a+\frac{b}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2),x)
[Out] x*ln(c*(a+b/x)^2)-2*b/a*ln(1/x)+2*b/a*ln(a+b/x)
```

maxima [A] time = 0.99, size = 28, normalized size = 1.00

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)+\frac{2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(a*x + b)/a
```

mupad [B] time = 0.15, size = 28, normalized size = 1.00

$$x\,\ln\left(\frac{c\,{\left(b+a\,x\right)}^2}{x^2}\right)+\frac{2\,b\,\ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2),x)
```

```
[Out] x*log((c*(b + a*x)^2)/x^2) + (2*b*log(b + a*x))/a
```

sympy [A] time = 0.28, size = 26, normalized size = 0.93

$$x\log\left(\frac{c\left(ax+b\right)^{2}}{x^{2}}\right)+\frac{2b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2),x)
```

```
[Out] x*log(c*(a*x + b)**2/x**2) + 2*b*log(a*x + b)/a
```

3.97 $\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=67

$$x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{8b\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

```
[Out] -4*b*ln(b/(a*x+b))*ln(c*(a*x+b)^2/x^2)/a+x*ln(c*(a*x+b)^2/x^2)^2+8*b*polylo
g(2,1-b/(a*x+b))/a
```

Rubi [A] time = 0.16, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/
x^2]^2 + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rubi steps

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + (4b)\int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax} dx$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)} dx, x, b +\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x} dx, x, \frac{}{b+}\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}} dx, x, \frac{1}{b+ax}\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 1.58

$$x\log^2\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} + \frac{8b\mathrm{Li}_2\left(\frac{b+ax}{b}\right)}{a} - \frac{4b\log^2\left(\frac{b}{ax+b}\right)}{a} - \frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)/a - (4
*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/x^2
]^2 + (8*b*PolyLog[2, (b + a*x)/b])/a
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\log\left(\frac{a^2cx^2 + 2\,abcx + b^2c}{x^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{(ax+b)^2c}{x^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^2, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{(ax+b)^2c}{x^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

maxima [A] time = 1.05, size = 118, normalized size = 1.76

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)^2+\frac{4\,b\log(ax+b)\log\left(\frac{(ax+b)^2c}{x^2}\right)}{a}+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\left(c\log(ax+b)-c\right.}{}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2)^2,x)
```

```
[Out] int(log((c*(b + a*x)^2)/x^2)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$4b\int\frac{\log\left(a^{2}c+\frac{2abc}{x}+\frac{b^{2}c}{x^{2}}\right)}{ax+b}\,dx+x\log\left(\frac{c\,(ax+b)^{2}}{x^{2}}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**2,x)
```

```
[Out] 4*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)/(a*x + b), x) + x*log(c*
(a*x + b)**2/x**2)**2
```

3.98 $\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=102

$$\frac{24b\text{Li}_2\left(\frac{ax}{b+ax}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*(a*x+b)^2/x^2)^3-6*b*ln(c*(a*x+b)^2/x^2)^2*ln(1-a*x/(a*x+b))/a+24*b*
ln(c*(a*x+b)^2/x^2)*polylog(2,a*x/(a*x+b))/a+48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{PolyLog}\left(3,1-\frac{b}{ax+b}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{c(\ldots}{}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, 1 - b/(b + a*x)])/a +
(48*b*PolyLog[3, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)dx = x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) + (6b)\int \frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}dx$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(24b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{x(b+ax)}dx}{a}$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 0.96

$$\frac{24b\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a} + \frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)])/a +
(48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\frac{a^2cx^2 + 2\,abcx + b^2c}{x^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{(ax + b)^2 c}{x^2}\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^3, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{(ax + b)^2 c}{x^2}\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a*x+b)^2*c/x^2)^3,x)
```

```
[Out] int(ln((a*x+b)^2*c/x^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\left(2\,(ax + b)\log\,(ax + b)^3 + 3\left(ax\log(c) - 2\,ax\log(x)\right)\log\,(ax + b)^2\right)}{a} + \int \frac{ax\log(c)^3 + b\log(c)^3 - 8\,(ax + b)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(2*(a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - 2*a*x*log(x))*log(a*x + b)^
2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - 8*(a*x + b)*log(x)^3 + 12*(a*
x*log(c) + b*log(c))*log(x)^2 + 6*((log(c)^2 - 4*log(c))*a*x + b*log(c)^2 +
```

```
 4*(a*x + b)*log(x)^2 - 4*(a*x*(log(c) - 2) + b*log(c))*log(x))*log(a*x + b
) - 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2)^3,x)
```

```
[Out] int(log((c*(b + a*x)^2)/x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$6b\int\frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)^2}{ax+b}\,dx+x\log\left(\frac{c\,(ax+b)^2}{x^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**3,x)
```

```
[Out] 6*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)**2/(a*x + b), x) + x*log
(c*(a*x + b)**2/x**2)**3
```

3.99 $\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=28

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2486, 31}

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-(2b)\int \frac{1}{b+ax}\,dx\\ &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2],x]
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

fricas [A] time = 0.62, size = 41, normalized size = 1.46

$$\frac{ax\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)-2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")
[Out] (a*x*log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2)) - 2*b*log(a*x + b))/a
```

giac [A] time = 0.25, size = 29, normalized size = 1.04

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2\,b\log\left(|ax+b|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(abs(a*x + b))/a
```

maple [B] time = 0.12, size = 79, normalized size = 2.82

$$x\ln\left(\frac{\left(\frac{b}{ax+b}-1\right)^2c}{a^2}\right)+\frac{2b\ln\left(\frac{1}{ax+b}\right)}{a}+\frac{b\ln\left(\frac{\left(\frac{b}{ax+b}-1\right)^2c}{a^2}\right)}{a}-\frac{2b\ln\left(\frac{b}{ax+b}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2),x)
[Out] ln(c*(b/(a*x+b)-1)^2/a^2)*x-2/a*b*ln(b/(a*x+b)-1)+1/a*ln(c*(b/(a*x+b)-1)^2/
a^2)*b+2/a*b*ln(1/(a*x+b))
```

maxima [A] time = 0.89, size = 28, normalized size = 1.00

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2\,b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(a*x + b)/a
```

mupad [B] time = 0.15, size = 28, normalized size = 1.00

$$x\,\ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)-\frac{2\,b\,\ln(b+a\,x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2),x)
```

```
[Out] x*log((c*x^2)/(b + a*x)^2) - (2*b*log(b + a*x))/a
```

sympy [A] time = 0.30, size = 26, normalized size = 0.93

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2),x)
```

```
[Out] x*log(c*x**2/(a*x + b)**2) - 2*b*log(a*x + b)/a
```

3.100 $\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=67

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}+\frac{8b\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

[Out] x*ln(c*x^2/(a*x+b)^2)^2+4*b*ln(c*x^2/(a*x+b)^2)*ln(b/(a*x+b))/a+8*b*polylog(2,1-b/(a*x+b))/a

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c*x^2)/(b + a*x)^2]^2,x]

[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 + (4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x)])/a + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rubi steps

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)dx = x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) - (4b)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}dx$$
$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}dx}{a}$$
$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}dx, x, b+ax\right)}{a^2}$$
$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}dx, x, \frac{1}{b+ax}\right)}{a^2}$$
$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}dx, x, \frac{1}{b+ax}\right)}{a^2}$$
$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{8b\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] time = 0.01, size = 106, normalized size = 1.58

$$x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}+\frac{8b\text{Li}_2\left(\frac{b+ax}{b}\right)}{a}-\frac{4b\log^2\left(\frac{b}{ax+b}\right)}{a}-\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 - (8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a + (
4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)
/a + (8*b*PolyLog[2, (b + a*x)/b])/a
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(a*x + b)^2)^2, x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c\,x^2}{(ax+b)^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

maxima [A] time = 1.12, size = 118, normalized size = 1.76

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2 - \frac{4\,b\log(ax+b)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + \frac{4\left(\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\left(c\log(ax+b)-c\log\right.}{a}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2)^2,x)
```

```
[Out] int(log((c*x^2)/(b + a*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-4b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**2,x)
```

```
[Out] -4*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))/(a*x + b), x) + x*lo
g(c*x**2/(a*x + b)**2)**2
```

3.101 $\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=98

$$\frac{24b\text{Li}_2\left(\frac{ax}{b+ax}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}+x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{48b\text{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)^3+6*b*ln(c*x^2/(a*x+b)^2)^2*ln(b/(a*x+b))/a+24*b*ln(c
*x^2/(a*x+b)^2)*polylog(2,a*x/(a*x+b))/a-48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{48b\text{PolyLog}\left(3,1-\frac{b}{ax+b}\right)}{a}+x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, 1 - b/(b + a*x)])/a - (
48*b*PolyLog[3, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) - (6b)\int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}dx$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{\left(24b^2\right)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}dx}{a}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a} - \frac{(4}{}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a} - \frac{4}{}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.00

$$\frac{24b\text{Li}_2\left(\frac{ax}{b+ax}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + x\log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b\text{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="fricas")
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="giac")
[Out] integrate(log(c*x^2/(a*x + b)^2)^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c\,x^2}{(ax+b)^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/(a*x+b)^2*c*x^2)^3,x)
[Out] int(ln(1/(a*x+b)^2*c*x^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(2\,(ax+b)\log(ax+b)^3-3\left(ax\log(c)+2\,ax\log(x)\right)\log(ax+b)^2\right)}{a}-\int -\frac{ax\log(c)^3+b\log(c)^3+8\,(ax+b)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="maxima")
[Out] -4*(2*(a*x + b)*log(a*x + b)^3 - 3*(a*x*log(c) + 2*a*x*log(x))*log(a*x + b)
^2)/a - integrate(-(a*x*log(c)^3 + b*log(c)^3 + 8*(a*x + b)*log(x)^3 + 12*(
a*x*log(c) + b*log(c))*log(x)^2 - 6*((log(c)^2 + 4*log(c))*a*x + b*log(c)^2
```

```
+ 4*(a*x + b)*log(x)^2 + 4*(a*x*(log(c) + 2) + b*log(c))*log(x))*log(a*x +
b) + 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2)^3,x)
```

```
[Out] int(log((c*x^2)/(b + a*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-6b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**3,x)
```

```
[Out] -6*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))**2/(a*x + b), x) + x
*log(c*x**2/(a*x + b)**2)**3
```

3.102 $\int \frac{\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx$

Optimal. Leaf size=35

$$-\frac{\text{Li}_3\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[Out] -polylog(3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {6610}

$$-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x]
```

```
[Out] -(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx = -\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.86

$$\frac{\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{ad-bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x
]
```

```
[Out] PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{Li}_2\left(\frac{bc-ad}{bdx+ad}+1\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

```
[Out] integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)*
x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

maple [A] time = 0.05, size = 36, normalized size = 1.03

$$\frac{\text{polylog}\left(3,-\frac{ad-bc}{(bx+a)d}+1\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/(a*d-b*c)*polylog(3,1-(a*d-b*c)/d/(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{polylog}\left(2,1-\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{2}\left(-\frac{ad}{ad+bdx}+\frac{bc}{ad+bdx}+1\right)}{(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(2, -a*d/(a*d + b*d*x) + b*c/(a*d + b*d*x) + 1)/((a + b*x)*
(c + d*x)), x)
```

3.103 $\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{Li}_3\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)-polylog(
3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2506, 6610}

$$\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/((a +
b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b
*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} + \int \frac{\operatorname{Li}_2\left(1-\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} - \frac{\operatorname{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.80

$$\frac{\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)-\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/
((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - Pol
yLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(-\frac{bc-ad}{bdx+ad}\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral(log(-(b*c - a*d)/(b*d*x + a*d))*log((d*e*x + c*e)/(b*x + a))/(b*d*
x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{(dx+c)e}{bx+a}\right)\ln\left(\frac{ad-bc}{(bx+a)d}\right)}{(bx+a)(dx+c)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a*d-b*c)/(b*x+a)/d)*ln((d*x+c)/(b*x+a)*e)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln((a*d-b*c)/(b*x+a)/d)*ln((d*x+c)/(b*x+a)*e)/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx}{2\,(ad-bc)} + \frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2ad-2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] b*Integral(log(c*e/(a + b*x) + d*e*x/(a + b*x))**2/(a + b*x), x)/(2*(a*d -
b*c)) + log((a*d - b*c)/(d*(a + b*x)))*log(e*(c + d*x)/(a + b*x))**2/(2*a*d
 - 2*b*c)
```

3.104 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$

Optimal. Leaf size=140

$$-\frac{2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}-\frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}+\frac{2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/b-2*ln
((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2488, 2506, 6610}

$$-\frac{2\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}+\frac{2\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/b) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (2*PolyLog[3, 1 + (b*c -
 a*d)/(d*(a + b*x))])/b
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
```

```
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{(2(bc-ad))\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{b}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b} - \frac{(2(bc-$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b} + \frac{2\mathrm{Li}_3\left(1\right.}{}$$

Mathematica [A] time = 0.04, size = 135, normalized size = 0.96

$$\frac{-2\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)-\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)+2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a
+ b*x))])/b
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\frac{bce-acf+\left(bde-adf\right)x}{ade-acf+\left(bde-bcf\right)x}\right)^2}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(b*x + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 879, normalized size = 6.28

$$-\frac{cf\ln\left(1-\frac{\left(bcf-bde\right)\left(\frac{\left(af-be\right)d}{\left(cf-de\right)b}-\frac{\left(af-be\right)\left(ad-bc\right)}{\left(cf-de\right)\left(bx+a\right)b}\right)}{adf-bde}\right)\ln\left(\frac{\left(af-be\right)d}{\left(cf-de\right)b}-\frac{\left(af-be\right)\left(ad-bc\right)}{\left(cf-de\right)\left(bx+a\right)b}\right)^{2}}{bcf-bde}+\frac{de\ln\left(1-\frac{\left(bcf-bde\right)\left(\frac{\left(af-be\right)d}{\left(cf-de\right)b}-\frac{\left(af-be\right)\left(ad-bc\right)}{\left(cf-de\right)\left(bx+a\right)b}\right)}{adf-bde}\right)\ln\left(\right.}{bcf-bde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x)
```

```
[Out] -1/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a
*f-b*e)/b)^2*ln(1-(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-
d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+1/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a
*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1-(b*c*f-b*d*e)/(
a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e
)/b))*d*e-2/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b)*polylog(2,(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-
b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+2/(b*c*f-b*d*e)*ln(-
(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,
(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b))*d*e+2/(b*c*f-b*d*e)*polylog(3,(b*c*f-b*d*e)/(a*d*f-b*d
*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f
-2/(b*c*f-b*d*e)*polylog(3,(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c
)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*d*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(dx+c)^3}{a} - \int -\frac{\left(\log\left(-be+af\right)^2 - 2\,\log\left(-be+af\right)\log\left(-de+cf\right) + \log\left(-de+cf\right)^2\right)bdx + \left(\log\left(-be+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="maxima")
```

```
[Out] log(d*x + c)^3/a - integrate(-((log(-b*e + a*f)^2 - 2*log(-b*e + a*f)*log(-
d*e + c*f) + log(-d*e + c*f)^2)*b*d*x + (log(-b*e + a*f)^2 - 2*log(-b*e + a
*f)*log(-d*e + c*f) + log(-d*e + c*f)^2)*b*c + (b*d*x + b*c)*log(b*x + a)^2
 - 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f)) + b*c*(log(-b*e + a*f) - lo
g(-d*e + c*f)))*log(b*x + a) + 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f))
 + b*c*(log(-b*e + a*f) - log(-d*e + c*f)) - (2*b*d*x + b*c + a*d)*log(b*x
+ a))*log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{\left(a\,f-b\,e\right)\,\left(c+d\,x\right)}{\left(c\,f-d\,e\right)\,\left(a+b\,x\right)}\right)^2}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(a + b*x), x)
```

3.105 $\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}-\frac{\text{Li}_3\left(\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}$$

[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)-polylog(3,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2506, 6610}

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

[In] Int[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]

[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))]/(b*c - a*d)

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \int \frac{\text{Li}_2\left(1-\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\text{Li}_3\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

Mathematica [A] time = 0.03, size = 96, normalized size = 0.88

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) - \text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*
e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))] - PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
])/(b*c - a*d)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(-\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+\left(bde-bcf\right)x}\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b
*c*f)*x))*log((d*e*x + c*e)/(b*x + a))/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)\left(fx+e\right)}{\left(de-cf\right)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)*(f*x + e)/((d*e - c*f
)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{(dx+c)e}{bx+a}\right)\ln\left(\frac{(ad-bc)(fx+e)}{(-cf+de)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((d*x+c)/(b*x+a)*e)*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x)
```

```
[Out] int(ln((d*x+c)/(b*x+a)*e)*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\log(bx+a)^2 - 2\left(\log(bx+a) - \log(e)\right)\log(dx+c) + \log(dx+c)^2 - 2\,\log(bx+a)\log(e)\right)\log\left(fx+e\right)}{2\,(bc-ad)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - log(e))*log(d*x + c) + log(d*x + c
)^2 - 2*log(b*x + a)*log(e))*log(f*x + e)/(b*c - a*d) + integrate(1/2*(2*(e
*log(-b*c + a*d)*log(e) - e*log(d*e - c*f)*log(e))*b*c + (b*d*f*x^2 + 2*b*c
*e - (2*d*e - c*f)*a + (3*b*c*f - a*d*f)*x)*log(b*x + a)^2 - 2*(d*e*log(-b*
c + a*d)*log(e) - d*e*log(d*e - c*f)*log(e))*a + 2*((f*log(-b*c + a*d)*log(
e) - f*log(d*e - c*f)*log(e))*b*c - (d*f*log(-b*c + a*d)*log(e) - d*f*log(d
*e - c*f)*log(e))*a)*x - 2*(b*d*f*x^2*log(e) - (e*(log(d*e - c*f) - log(e))
 - e*log(-b*c + a*d))*b*c + (d*e*(log(d*e - c*f) - log(e)) - d*e*log(-b*c +
 a*d) + c*f*log(e))*a + ((f*log(-b*c + a*d) - f*log(d*e - c*f) + 2*f*log(e)
)*b*c - (d*f*log(-b*c + a*d) - d*f*log(d*e - c*f))*a)*x)*log(b*x + a) + 2*(
b*d*f*x^2*log(e) + (e*log(-b*c + a*d) - e*log(d*e - c*f))*b*c - (d*e*log(-b
*c + a*d) - d*e*log(d*e - c*f) - c*f*log(e))*a + ((f*log(-b*c + a*d) - f*lo
g(d*e - c*f) + f*log(e))*b*c - (d*f*log(-b*c + a*d) - (f*log(d*e - c*f) + f
*log(e))*d)*a)*x - (b*d*f*x^2 + 2*b*c*f*x + b*c*e - (d*e - c*f)*a)*log(b*x
+ a))*log(d*x + c))/(a*b*c^2*e - a^2*c*d*e + (b^2*c*d*f - a*b*d^2*f)*x^3 -
(a*b*d^2*e + a^2*d^2*f - (c*d*e + c^2*f)*b^2)*x^2 + (b^2*c^2*e + a*b*c^2*f
- (d^2*e + c*d*f)*a^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(-\frac{\left(e+f\,x\right)\,(a\,d-b\,c)}{\left(c\,f-d\,e\right)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)^2\log\left(\frac{\left(e+fx\right)(ad-bc)}{(a+bx)\left(-cf+de\right)}\right)}{2ad-2bc}-\frac{\left(af-be\right)\int\frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{ae+afx+bex+bfx^2}\,dx}{2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b
*x+a)/(d*x+c),x)
```

```
[Out] log(e*(c + d*x)/(a + b*x))**2*log((e + f*x)*(a*d - b*c)/((a + b*x)*(-c*f +
d*e)))/(2*a*d - 2*b*c) - (a*f - b*e)*Integral(log(c*e/(a + b*x) + d*e*x/(a
+ b*x))**2/(a*e + a*f*x + b*e*x + b*f*x**2), x)/(2*(a*d - b*c))
```

3.106 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{2\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}-\frac{2\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}-\frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

```
[Out] -ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+
d*e)/(b*x+a))/(-a*f+b*e)-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylo
g(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)+2*polylog(3,(-a*f+b*e
)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)
```

Rubi [A] time = 0.25, antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2503, 2506, 6610}

$$\frac{2\text{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{be-af}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{be-af}-\frac{\log\left(-\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx}{(a+bx}\right.}{be-af}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e +
f*x)),x]
```

```
[Out] -((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[-(((b*c - a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x)))])/(b*e - a*f)) - (2*Log[((b*e - a*f)*
(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/
((d*e - c*f)*(a + b*x))])/(b*e - a*f) + (2*PolyLog[3, 1 + ((b*c - a*d)*(e +
 f*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{(2(bc-ad))\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}$$

Mathematica [B] time = 0.52, size = 1636, normalized size = 8.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)
*(e + f*x)),x]
```

```
[Out] (-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/d
 + x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c/d
 + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*Log
[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*Log[a/
b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] -
```

```
6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b
*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Lo
g[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a + b*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b + x]^2*Log[e
+ f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d + x]^2*Log[e
+ f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
 + b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*
x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 6*L
og[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)
] + 3*Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] +
6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*
(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 3*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] + 3
*Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 6*Log[a/b + x]*Log[(f*(c +
 d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[(f*(c + d*x))
/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 6*Log[c/d + x]*Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
 + 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)
*(a + b*x))] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*
(Log[a/b + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 6*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 6*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] + 6*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(3*b*e
- 3*a*f)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\frac{bce-acf+\left(bde-adf\right)x}{ade-acf+\left(bde-bcf\right)x}\right)^2}{bfx^2+ae+\left(be+af\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(b*f*x^2 + a*e + (b*e + a*f)*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.05, size = 357, normalized size = 1.75

$$\frac{\ln\left(\frac{(af-be)d}{(cf-de)b}-\frac{(af-be)(ad-bc)}{(cf-de)(bx+a)b}\right)^{2}\ln\left(-\frac{(af-be)d}{(cf-de)b}+\frac{(af-be)(ad-bc)}{(cf-de)(bx+a)b}+1\right)}{af-be}+\frac{2\,\mathrm{polylog}\left(2,\frac{(af-be)d}{(cf-de)b}-\frac{(af-be)(ad-bc)}{(cf-de)(bx+a)b}\right)\ln\left(\frac{(af-be)}{(cf-de)}\right.}{af-be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x)
```

```
[Out] 1/(a*f-b*e)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x
+a)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b-1/(c*f-d*e)*(a*f-b*e)
/b*d)+2/(a*f-b*e)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e
)/(b*x+a)/b)*polylog(2,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d
*e)/(b*x+a)/b)-2/(a*f-b*e)*polylog(3,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a
*d-b*c)/(c*f-d*e)/(b*x+a)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{\left(a\,f-b\,e\right)\,\left(c+d\,x\right)}{\left(c\,f-d\,e\right)\,\left(a+b\,x\right)}\right)^2}{\left(e+f\,x\right)\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{(a+bx)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a)/(f*x+e),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/((a + b*x)*(e + f*x)), x)
```

3.107 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$

Optimal. Leaf size=322

$$-\frac{2\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}-\frac{2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+\frac{2\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log}{f}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/f+ln((
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/
(b*x+a))/f-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/
d/(b*x+a))/f+2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*
e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f+2*polylog(3,b*(d*x+c)/d/(b*x+a))/f-2*polyl
og(3,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f
```

Rubi [A] time = 0.51, antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2489, 2488, 2506, 6610, 2503}

$$-\frac{2\text{PolyLog}\left(3,1-\frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{f}+\frac{2\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*
(a + b*x))]^2)/f) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2
*Log[((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/f + (2*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 - (b*c - a*d)/(b*(c +
d*x))])/f - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog
[2, 1 - ((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/f + (2*PolyLog[3,
 1 - (b*c - a*d)/(b*(c + d*x))])/f - (2*PolyLog[3, 1 - ((b*c - a*d)*(e + f*
x))/((b*e - a*f)*(c + d*x))])/f
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

```
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \frac{d\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}\,dx}{f} - \frac{(de-cf)\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)}\,dx}{f}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} - \frac{(2(bc-a}{}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2\log\left(\frac{(b}{(d}\right.}{}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2\log\left(\frac{(b}{(d}\right.}{}$$

Mathematica [B] time = 0.28, size = 1080, normalized size = 3.35

$$\frac{\log(e+fx)\log^2\left(\frac{a}{b}+x\right) - \log\left(\frac{b(e+fx)}{be-af}\right)\log^2\left(\frac{a}{b}+x\right) - 2\log\left(\frac{c}{d}+x\right)\log(e+fx)\log\left(\frac{a}{b}+x\right) + 2\log\left(\frac{(be-af)(c+dx}{(de-cf)(a+bx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) + Log[a/b + x]^2*Log[e + f*x] - 2*Log[a/b + x]*Log[c/d +
x]*Log[e + f*x] + Log[c/d + x]^2*Log[e + f*x] + 2*Log[a/b + x]*Log[((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] - 2*Log[c/d + x]*Log[
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] + Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[e + f*x] - Log[a/b + x]^2*L
og[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) +
c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*
Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))
/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[(f*(c + d*
x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Lo
g[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[c/d + x]*Log
[(d*(e + f*x))/(d*e - c*f)] - Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)]
 - 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e
- c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)
] + 2*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log
```

```
[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
+ Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*
PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{bce-acf+\left(bde-adf\right)x}{ade-acf+\left(bde-bcf\right)x}\right)^2}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(f*x + e), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 4733, normalized size = 14.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x)
```

```
[Out] 2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*
e)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(
1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*d^2*e*a^
```

```
2+b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/
(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f
-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*d^2*e*a^2-2/
(a*f-b*e)/f/(a*d-b*c)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f
-d*e)/(b*x+a)/b)*polylog(2,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c
*f-d*e)/(b*x+a)/b)*a*b*d*e-2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylo
g(3,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d
-b*c)/(c*f-d*e)/(b*x+a)/b))*d*e^2*c-1/(a*f-b*e)/f/(a*d-b*c)*ln(1/(c*f-d*e)*
(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*ln(-1/(c*f-d*e)*(a
*f-b*e)/b*d+(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b+1)*a*b*d*e+2/(a*f-b*e)/
f/(a*d-b*c)*polylog(3,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*
e)/(b*x+a)/b)*a*b*d*e-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c
*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*
f-d*e)/(b*x+a)/b))*d^2*e*a^2+2/(a*f-b*e)/f/(a*d-b*c)*ln(1/(c*f-d*e)*(a*f-b*
e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*polylog(2,1/(c*f-d*e)*(a*f-
b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*b^2*c*e-2*b^3/(b*c*f-b*d*
e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)
*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*e)*(a*
f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*c^2*e-2*b^2/(b*c*f-b*d*
e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)
*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*c^2*a*f-b^3/(b*c*f
-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*
(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*e)*(a*f
-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*c^2*e+1/(a*f-b*e)/f/(a
*d-b*c)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/
b)^2*ln(-1/(c*f-d*e)*(a*f-b*e)/b*d+(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b+
1)*b^2*c*e-2/(a*f-b*e)/(a*d-b*c)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*
d-b*c)/(c*f-d*e)/(b*x+a)/b)*polylog(2,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(
a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*a*b*c-1/(a*f-b*e)/(a*d-b*c)*ln(1/(c*f-d*e)*(a
*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*ln(-1/(c*f-d*e)*(a*f
-b*e)/b*d+(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b+1)*a*b*c-2/(a*f-b*e)/f/(a
*d-b*c)*polylog(3,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(
b*x+a)/b)*b^2*c*e+2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f
-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-
d*e)/(b*x+a)/b))*c^2*e+1/(a*f-b*e)/(a*d-b*c)*ln(1/(c*f-d*e)*(a*f-b*e)/b*d-(
a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*ln(-1/(c*f-d*e)*(a*f-b*e)/b*d+(a*
f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b+1)*a^2*d+2/(a*f-b*e)/(a*d-b*c)*ln(1/(c
*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*polylog(2,1/
(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*a^2*d+2/(a
*f-b*e)/(a*d-b*c)*polylog(3,1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(
c*f-d*e)/(b*x+a)/b)*a*b*c-2/(a*f-b*e)/(a*d-b*c)*polylog(3,1/(c*f-d*e)*(a*f-
b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*a^2*d+b^2/(b*c*f-b*d*e)/(
a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-b*e)
/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*e)*(a*f-b*e)/b*d
-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*c^2*a*f+2*b^2/(b*c*f-b*d*e)/(a*
```

```
f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-
b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*e)*(a*f-b*e)
/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*c^2*a*f+2*b/(b*c*f-b*d*e)/(a*
f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f-d*e)*(a*f-
b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*c*a^2*d*f+2*b^2/(b*c*f-b
*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*f
-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*d^2*e^2*a+b^3
/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(
c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-
d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*d*e^2*c-b^2/(
b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*
f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*
e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*d^2*e^2*a+2*b^3
/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e
)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1
/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*d*e^2*c-b
/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(1/(c*
f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1/(c*f-d*
e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)^2*c*a^2*d*f-2*b^2
/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e
)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln(1
/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*d^2*e^2*a
-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d
*e)*(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b))*ln
(1/(c*f-d*e)*(a*f-b*e)/b*d-(a*f-b*e)*(a*d-b*c)/(c*f-d*e)/(b*x+a)/b)*c*a^2*d
*f
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{\left(a\,f-b\,e\right)\,\left(c+d\,x\right)}{\left(c\,f-d\,e\right)\,\left(a+b\,x\right)}\right)^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{e+fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(e + f*x), x)
```

3.108 $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=433

$$-\frac{\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}-\frac{\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}+\frac{\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+}{(a+}\right.}{2(bc-ad)}$$

```
[Out] -1/2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(-
a*d+b*c)-1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(b*(f*x+e)/(-a*f
+b*e))/(-a*d+b*c)+1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*
f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)-ln((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)+ln((-a*f+b*e)*(d*x+c
)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*
d+b*c)+polylog(3,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)-polylog(3,(-a*f+b*e)*(d*x+
c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] time = 0.59, antiderivative size = 445, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 6, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.092, Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$-\frac{\text{PolyLog}\left(3,1-\frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{bc-ad}+\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}-\frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,1-\frac{(e-}{(c-}\right.}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))
/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] -(Log[(b*c - a*d)/(b*(c + d*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]^2)/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((b*c - a*d)*(e + f*x))/((b
*e - a*f)*(c + d*x))])/(2*(b*c - a*d)) + (Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d)
- (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 - ((b*
c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/(b*c - a*d) + PolyLog[3, 1 -
(b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) - PolyLog[3, 1 - ((b*c - a*d)*(e + f
*x))/((b*e - a*f)*(c + d*x))]/(b*c - a*d)
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
```

```
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
```

```
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{f\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx}{2(bc-ad)}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{d\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}\,dx}{2(bc-ad)} - \frac{(de-cf)\int \cdots}{2(b\cdots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log\cdots}{\cdots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log\cdots}{\cdots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log\cdots}{\cdots}$$

Mathematica [B] time = 0.82, size = 908, normalized size = 2.10

$$\frac{\log\left(\frac{b(e+fx)}{be-af}\right)\log^2(c+dx) - \log\left(\frac{d(e+fx)}{de-cf}\right)\log^2(c+dx) - 2\log(a+bx)\log\left(\frac{b(e+fx)}{be-af}\right)\log(c+dx) - 2\log\left(\frac{(be-af)(c+d\cdots}{(de-cf)(a+b\cdots}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e +
 f*x))/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[a + b*x]*Log[c + d*x]*Log[(b*(e + f*x))/(b*e - a*
f)] + Log[c + d*x]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a + b*x]*Log[(f
```

```
*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*
x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[c + d*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*
f)] + 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c +
d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a +
 b*x]*Log[c + d*x]*Log[(d*(e + f*x))/(d*e - c*f)] - Log[c + d*x]^2*Log[(d*(
e + f*x))/(d*e - c*f)] - 2*Log[a + b*x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*L
og[(d*(e + f*x))/(d*e - c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*
(e + f*x))/(d*e - c*f)] + 2*Log[c + d*x]*Log[((b*e - a*f)*(c + d*x))/((d*e
- c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(
d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e
 + f*x))/(d*e - c*f)] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a +
b*x))] + 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2,
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x
))/(d*(a + b*x))] - 2*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
b*x))])/(2*b*c - 2*a*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\frac{bce-acf+\left(bde-adf\right)x}{ade-acf+\left(bde-bcf\right)x}\right)\log\left(\frac{bfx+be}{be-af}\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))*log((b*f*x + b*e)/(b*e - a*f))/(b*d*x^2 + a*c + (b*c + a*d)*x),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{\left(fx+e\right)b}{be-af}\right)\log\left(\frac{\left(be-af\right)(dx+c)}{\left(de-cf\right)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((f*x + e)*b/(b*e - a*f))*log((b*e - a*f)*(d*x + c)/((d*e - c*
f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 3.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{(fx+e)b}{-af+be}\right)\ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

```
[Out] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(-\frac{b\,(e+f\,x)}{a\,f-b\,e}\right)\,\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*
e)*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*
e)*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e)
)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```